International Macroeconomics 7: Dynamic International Macroeconomics and Open-Economy Policy*

Contents

*These lecture notes closely and sometimes literally follow sections from: Epstein, Brendan. *Masters Level International Macroeconomics*, © 2013.

1 Preliminaries

- These lecture notes deal with dynamic International Macroeconomics. We begin by examining a small open economy that exists for two periods, only. This context allows us to build intuition for the latter part of these notes, whose topic is International Real Business Cycle models—a context in which the economies involved are assumed to be large.

- The distinction between small and large open economies relates to whether they take as given or not the world interest rate. Small open economies are price takers. In contrast, large open economies' actions can indeed have an impact on the world interest rate.

2 Intertemporal Trade, the Current Account, and the Gains from Financial Openness

- In order to fix ideas more easily, before proceeding to the analysis of Open Economy (or International) Real Business Cycles models (which focus on large open economies), we'll first focus on a small open economy. In a small open economy, from the economy's viewpoint the real interest rate, r, is exogenous and determined in the world capital market. In essence, think about a small open economy as being analogous to perfect competition in Microeconomics from the economy's viewpoint. This means that the small open economy is a "small player" in world financial markets, so it acts as a price taker as far as the world real interest rate goes. (In contrast, in a large open economy context an economy understands that its actions can influence the world real interest rate.) All told, a small open economy can carry out any intertemporal exchange of consumption it desires at the given world interest rate r.

- Throughout this section, we ignore nominal prices and assume instead that all variables are in real terms. We assume that the economy is inhabited by a continuum of identical individuals with unit mass grouped into an aggregate risk-sharing household. As usual in a context such as this one, all non-price variables are normalized by the economy's population and all prices variables are normalized by the (world) price of final output.

- For simplicity, we assume that the economy exists for **two periods**, only, and we initially assume that there is no investment or government spending. Also for simplicity, we omit work/leisure and overall production considerations: instead, in each period some amount of output Y_t "falls from the sky" (this situation is usually referred to as being characteristic of an "endowment economy") with **certainty** (and the economy knows **exactly** how much output will be available in each period, meaning that there is "perfect foresight"). All told, these assumptions mean that we don't need expectation operators. Furthermore, we assume that the endowment is "perishable" in the sense that if nothing is done with a period's output, be that consuming it or lending it out, then that period's output is not available in the next period.

2.1 The Household

- The household maximizes lifetime utility:

$$\begin{aligned} U_t &= \sum\nolimits_{t=0}^{1} \beta^t u(C_t) \\ &= \beta^0 u(C_0) + \beta^1 u(C_1) \\ &= u(C_0) + \beta u(C_1), \end{aligned}$$

  where: U denotes lifetime utility; u is the instantaneous utility function; $\beta \in (0,1)$ is the (subjective) discount factor (recall that β measures the household's impatience to consume, and β is the Greek letter "beta"); furthermore, we assume typical properties for u: $u' > 0$, $u'' < 0$, which means that u is strictly concave. In this context, the household can be seen as maximizing lifetime utility subject to the **long-run budget constraint**.

- Let r_t denote the real interest rate for borrowing or lending in the world capital market that prevails over period $t-1$ (this includes the simplifying possibility that $r_t = r$ $\forall t$, where $\forall$ means "for all"). In **present value terms**, the household's lifetime budget constraint is (where the price of the output endowment—and therefore that of consumption—is normalized to 1 in all periods):

$$C_0 + \frac{C_1}{1+r_1} \leq Y_0 + \frac{Y_1}{1+r_1}.$$

  This lifetime budget constraint just says that lifetime discounted consumption must be less than or equal to lifetime discounted endowment.

- All told, the household's problem is to maximize lifetime utility subject to the lifetime budget constraint by choosing consumption in period 0 and consumption in period 1:

$$\max_{C_0,\, C_1} U_0 = u(C_0) + \beta u(C_1)$$

  such that:

$$C_0 + \frac{C_1}{1+r_1} \leq Y_0 + \frac{Y_1}{1+r_1}$$

  Because utility is increasing in consumption, the lifetime budget constraint will <u>bind</u>: all of the available endowment will be consumed. Notice that the intertemporal structure of the problem is such that once consumption in any one period is determined, then consumption in the other period is determined as well (since it is, in essence, a residual). Therefore, an approach to solving this problem, which aids in building intuition, is to solve for C_1 in terms of output in both periods, r_1 and C_0:

$$\begin{gathered} C_0 + \frac{C_1}{1+r_1} = Y_0 + \frac{Y_1}{1+r_1} \\ \rightarrow (1+r_1)\,C_0 + C_1 = (1+r_1)\,Y_0 + Y_1 \\ \text{(multiplying through by } 1+r_1\text{)} \\ \rightarrow C_1 = (1+r_1)\,(Y_0 - C_0) + Y_1. \end{gathered}$$

Substituting back into the objective function we can restate the household's problem as one in which **only consumption in period 0 is chosen**:

$$\max_{C_0} U_0 = u(C_0) + \beta u(\underbrace{(1+r_1)\,(Y_0 - C_0) + Y_1}_{=C_1(C_0,\cdot)\ (C_1 \text{ is a function of } C_0 \text{ and other things})}).$$

- The first-order condition (FOC) for this problem involves taking the partial derivative with respect to C_0, only, and setting it equal to zero (this FOC defines the level of C_0 for which lifetime utility is maximized and, implicitly, the corresponding level of C_1):

$$\begin{gathered} \frac{\partial U_0}{\partial C_0} \overset{!}{=} 0 \\ \rightarrow \frac{\partial\left[u(C_0) + \beta u(C_1(C_0))\right]}{\partial C_0} = 0 \\ \rightarrow \underbrace{\frac{\partial u(C_0)}{\partial C_0}}_{=u'(C_0)} + \beta \cdot \underbrace{\overbrace{\frac{\partial u(C_1)}{\partial C_1}}^{=u'(C_1)} \cdot \frac{\partial C_1}{\partial C_0}}_{\text{by the chain rule}} = 0 \\ \rightarrow u'(C_0) + \beta u'\left(C_1\right) \cdot \underbrace{\left[-\left(1+r_1\right)\right]}_{=\frac{\partial C_1}{\partial C_0} = \frac{\partial[(1+r_1)(Y_0 - C_0) + Y_1]}{\partial C_0}} = 0 \\ \rightarrow u'(C_0) = (1+r_1)\,\beta u'\left(C_1\right), \end{gathered}$$

which is the Euler equation in the present context.

2.2 Consumption Smoothing

- You've encountered consumption smoothing before, specifically in the textbook readings. Here, we will derive mathematically the condition under which consumption smoothing is in fact desirable and what the implications of this desirability are. In particular, consumption smoothing arises under the situation in which $\beta = 1/\left(1+r_1\right)$, meaning the the household discounts the future at the same rate that the market does.

Then, the Euler equation from the previous section can be rearranged as such:

$$\begin{aligned} u'(C_0) &= (1+r_1)\,\beta u'(C_1) \\ \rightarrow u'(C_0) &= (1+r_1)\cdot \underbrace{\frac{1}{1+r_1}}_{=\beta \text{ by assumption}} \cdot u'(C_1) \\ \rightarrow u'(C_1) &= u'(C_0), \end{aligned}$$

which implies that the economy **desires a flat lifetime consumption path**: $C_0 = C_1 = C$ (where C denotes this level of flat lifetime consumption).

- Using the budget constraint we can solve for this optimal level of C:

$$\begin{aligned} C_0 + \frac{C_1}{1+r_1} &= Y_0 + \frac{Y_1}{1+r_1} \\ \rightarrow C + \frac{C}{1+r_1} &= Y_0 + \frac{Y_1}{1+r_1} \\ &(\text{setting } C_1 = C_2 = C, \text{ which is optimal given } \beta = \frac{1}{1+r_1} \\ \rightarrow (1+r_1)\,C + C &= (1+r_1)Y_0 + Y_1 \\ \rightarrow (2+r_1)C &= (1+r_1)Y_0 + Y_1 \\ \rightarrow C &= \frac{(1+r_1)Y_0 + Y_1}{2+r_1}, \end{aligned}$$

which is the level of consumption that the household desires in each period.

2.2.1 What Happens if $Y_0 = Y_1 = Y$?

- If the endowment $Y_0 = Y_1 = Y$, i.e., the endowment is the same across periods, then the household's consumption smoothing path is given by construction. In this case, using the budget constraint:

$$\begin{aligned} C &= \frac{(1+r_1)Y_0 + Y_1}{2+r_1} \\ &= \frac{(1+r_1)Y + Y}{2+r_1} \\ &= \frac{2+r_1}{2+r_1}Y \\ &= Y. \end{aligned}$$

2.2.2 What Happens if $Y_0 < Y_1$?

- If the endowment $Y_0 < Y_1$, i.e., there is **less** endowment in the initial period compared to the following period, then to achieve consumption smoothing the economy can **borrow** the difference $C - Y_0 > 0$ from foreign residents ("foreigners" for short) on date 0 such that:

$$\begin{aligned} C_0 &= Y_0 + (C - Y_0) \\ &= C. \end{aligned}$$

Then, repayment with interest in period 1 implies that in this period the country has to repay:

$$(1 + r_1)(C - Y_0).$$

Therefore, by construction it must be the case that

$$\begin{aligned} C_1 &= Y_1 - (1 + r_1)(C - Y_0) \\ &= C. \end{aligned}$$

2.2.3 What Happens if $Y_0 > Y_1$?

- If the endowment $Y_0 > Y_1$, i.e., there is **more** endowment in the initial period compared to the following period, then to achieve consumption smoothing the economy can **loan** the difference $Y_0 - C > 0$ foreigners on date 0 (remember that the endowment is perishable) such that:

$$\begin{aligned} C_0 &= Y_0 - (Y_0 - C) \\ &= C. \end{aligned}$$

Then, repayment with interest in period 1 implies that in this period the country receives:

$$(1 + r_1)(Y_0 - C).$$

Therefore, by construction it must be the case that

$$\begin{aligned} C_1 &= Y_1 + (1 + r_1)(Y_0 - C) \\ &= C. \end{aligned}$$

2.2.4 What Happens if the Economy is Closed?

- Because output is perishable, then the only option that the household has if the economy is closed is $C_0 = Y_0$ and $C_1 = Y_1$.

2.3 The Current Account

- In the preceding development, the current account does not appear explicitly in the long-run budget constraint. But, it's there in the background. Recall that the current account balance over a period of time is the change in the value of a country's net claims on the rest of the world, that is, the change in a country's net foreign assets. Letting A_t stand for net foreign assets, then $dA_t = CA_t$, where CA is the current account. Then, by definition of national accounts,

$$GNDI_t = GNE_t + CA_t,$$

where: $GNDI$ is gross national disposable income; and GNE is gross national expenditure. Of course, $GNE_t = C_t + I_t + G_t$. In the current setup we don't have investment or government spending, so $GNE_t = C_t$. Also by definition of the current account

$$GNDI_t = GDP_t + NFIA_t + NUT_t,$$

where, recall: GDP is gross domestic product; $NFIA$ is net factor income from abroad (which is equal to the difference between foreign payments made to domestic entities for factor service exports—that is, foreign payments on capital, labor, and land owned by domestic entities—and factor service imports—that is, domestic payments made on capital, labor, and land owned by foreign entities); and NUT is net unilateral transfers (which is equal to the difference between gifts received by foreign entities to domestic entities and gifts made from domestic entities to foreign entities)

- In practice, NUT is very small in the national accounts, so we assume that $NUT_t = 0$ $\forall t$. Also, note that in this case $NFIA_t$ is simply equal to $r_t A_t$.

- So,

$$\begin{aligned} GNDI_t &= GNE_t + CA_t \\ \rightarrow Y_t + NFIA_t &= C_t + CA_t \\ \rightarrow Y_t + r_t A_t &= C_t + A_{t+1} - A_t \\ \rightarrow Y_t + (1 + r_t)A_t &= C_t + A_{t+1}. \end{aligned}$$

- Finally, note that in period t the product $(1+r_t)A_t$ is *predetermined*: r_t is the interest that prevailed on date $t-1$; and A_t is the value of the economy's net foreign assets at the end of period $t-1$. So, given the output endowment Y_t the economy's choice variables are explicitly C_t and implicitly A_{t+1} (A_t is an endogenous state variable).

- **Of course, the conceptual background for much of the preceding math lies in many of the textbook readings thus far**.

2.4 Graphical Optimization in the Two-Period Model

- The nice thing about focusing on a two-period model is that it lends itself for building intuition via graphical representation.

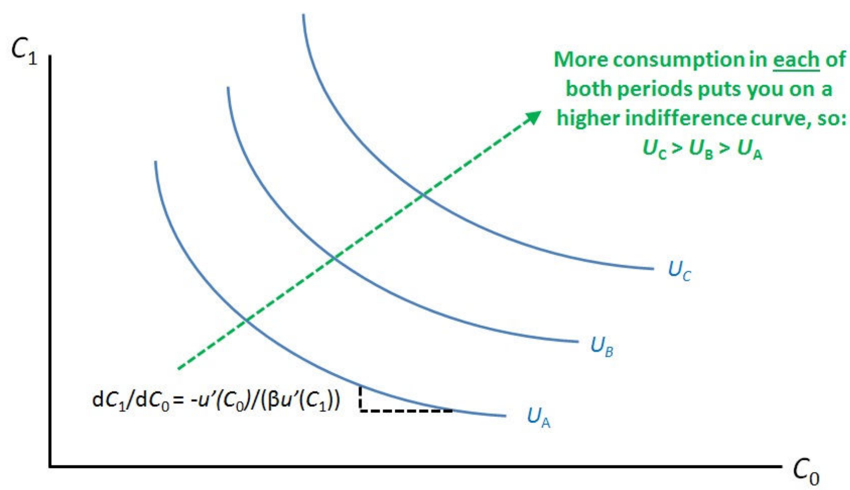
- Recall that an indifference curve denotes all combinations of consumption that yield the same utility. In the present intertemporal framework, the economy cares about maximizing (discounted) lifetime utility:

$$U_0 = u(C_0) + \beta u(C_1)$$
$$\rightarrow dU_0 = u'(C_0)dC_0 + \beta u'(C_1)dC_1$$
(taking the total derivative of lifetime utility).

On an indifference curve, $dU_0 = 0$; therefore on an indifference curve

$$u'(C_0)dC_0 + \beta u'(C_1)dC_1 = 0$$
$$\rightarrow \frac{dC_1}{dC_0} = -\frac{u'(C_0)}{\beta u'(C_1)}$$

is the slope of an indifference curve in (C_0, C_1) space. Thus:



If you think about it this graph is pretty interesting. Just like in Microeconomics the benchmark utility maximization problem involves choosing two distinct goods, say X and Y, and you can represent preferences over these two goods in, say, (Y, X) space. In the present context we are getting a two-dimensional representation of preferences over the single consumption good across time. Keep in mind, though, that just like in Microeconomics an "umbrella in sunny weather" is a **distinct** good from an "umbrella under rainy conditions," in this intertemporal context consumption in period 0 is in fact a different good than consumption in period 1.

- Now, let's work with the economy's net foreign assets position in each period to go back to the budget constraint we were working with earlier. In the two-period model, $A_2 = 0$ is optimal (transversality condition). Furthermore, let's **assume** $A_0 = 0$ (no international claims at the time the economy is "born"—recall that in any period t, A_t is predetermined, that is, inherited from last period). Therefore, the period-0 budget constraint is:

$$
\begin{aligned}
C_0 + A_1 &= Y_0 + (1+r_0)\cdot \underbrace{A_0}_{=0 \text{ by assumption}} \\
\rightarrow C_0 + A_1 &= Y_0 \\
\rightarrow A_1 &= Y_0 - C_0,
\end{aligned}
$$

and the period-1 budget constraint is:

$$
\begin{aligned}
C_1 + \underbrace{A_2}_{=0 \text{ by transversality condition}} &= Y_1 + (1+r_1)A_1 \\
\rightarrow C_1 &= Y_1 + (1+r_1)A_1.
\end{aligned}
$$

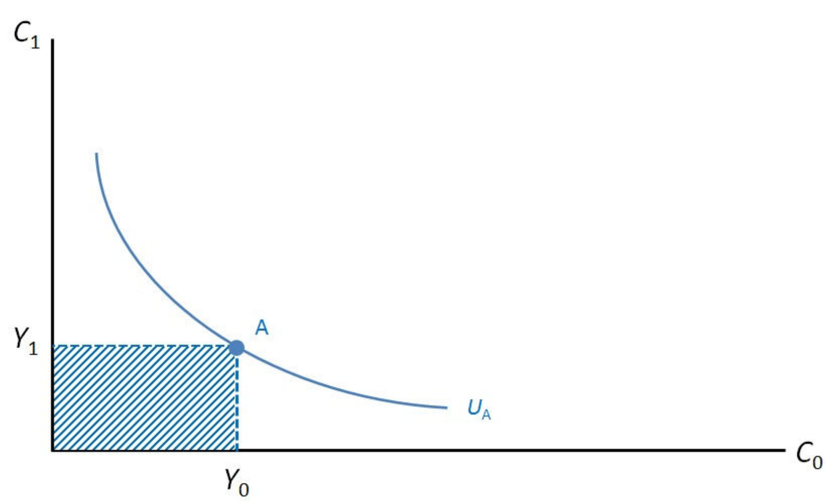
Because the period-0 constraint implies that that $A_1 = Y_0 - C_0$, then plugging into the period-1 constraint we have:

$$
\begin{aligned}
C_1 &= Y_1 + (1+r_1)(Y_0 - C_0) \\
\rightarrow C_1 &= Y_1 - (1+r_1)(C_0 - Y_0), \qquad (1)
\end{aligned}
$$

which is the economy's effective budget constraint when it is open.

2.4.1 What Happens if the Economy is Closed?

- In "autarky," that is, when the economy is closed, because the output endowment is perishable, then as noted earlier it must be the case that $C_0 = Y_0$ and $C_1 = Y_1$. Because instantaneous utility is increasing and concave in consumption, then in this case of autarky the economy is consuming at point A, and the economy is on indifference curve U_A (**the hatched box below is the economy's effective constraint under autarky: it can consume any point within that box, but it is optimal to be on the upper right-hand side corner because, there, all of the endowment is consumed in each period**):



- It is important to note that the optimality condition

$$\frac{\beta u'(C_1)}{u'(C_0)} = \frac{1}{1+r}$$

must hold whether the economy is open or closed. When the economy is open, the household takes as given the world interest rate r_1 and chooses consumption across periods so that

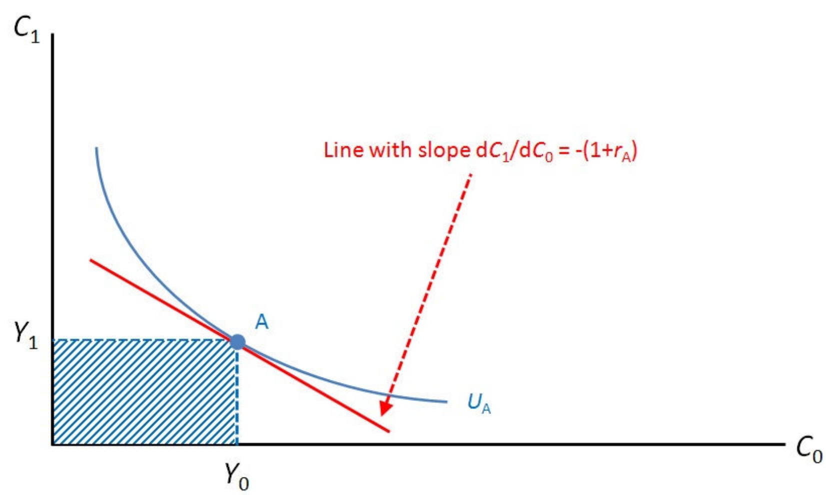
$$\frac{\beta u'(C_1)}{u'(C_0)} = \frac{1}{1+r_1}$$

holds, **meaning that <u>given</u> the right-hand side of this equation the <u>left-hand</u> side is determined**. When the economy is closed, it is unavoidably the case $C_1 = Y_1$ and $C_0 = Y_0$, so in this case

$$\frac{\beta u'(Y_1)}{u'(Y_0)} = \frac{1}{1+r_A},$$

where r_A denotes the "autarky interest rate" and the **<u>right-hand</u> side of this equation is determined <u>given</u> its left-hand side**. In essence the autarky interest rate is the interest rate that implicitly prevails at home whether the economy is open or not, and therefore, the interest rate that the household compares to the world interest rate when deciding whether or not to engage in international financial transactions and, if so, what international financial transactions to engage in (more details follow later below).

- All told, in (C_0,C_1) space, at the autarky consumption point (Y_0, Y_1) there is a line with slope $-(1 + r^A)$ that is tangent to the autarky indifference curve reflecting an implicit optimality condition (akin to the standard analogous tangency condition in Microeconomics):



- Finally, suppose that $\beta = 1/(1+r_1)$, which is the case in which consumption smoothing is desired. Then, the optimality condition under autarky that defines the autarky real interest rate,

$$\frac{1}{1+r^A} = \frac{\beta u'(Y_1)}{u'(Y_0)}$$
$$\text{(by } C_0 = Y_0 \text{ and } C_1 = Y_1\text{)}$$

implies that:

$$\frac{1}{1+r^A} = \frac{1}{1+r_1}\frac{u'(Y_1)}{u'(Y_0)}.$$

So, in this special case in which $\beta = 1/(1+r_1)$ the only reason that $r^A \neq r_1$ is because $Y_0 \neq Y_1$.

2.4.2 What Happens if the Economy is Open?

- Recall that the open economy's effective budget constraint is given by equation (1). So, when the economy is open it can consume any point **at** or **within** this budget constraint, which can be stated mathematically as:

$$C_1 \leq Y_1 - (1+r_1)(C_0 - Y_0).$$

In turn, the **budget line** satisfies:

$$C_1 = Y_1 - (1+r_1)(C_0 - Y_0).$$

- Let's find the intercepts of the (open-economy) budget line. If $C_0 = 0$ then the budget line implies that:

$$C_1 = Y_1 - (1+r_1)(0 - Y_0)$$
$$\rightarrow C_1 = Y_1 + (1+r_1)Y_0.$$

If, instead, $C_1 = 0$ then the economy's budget line implies that:

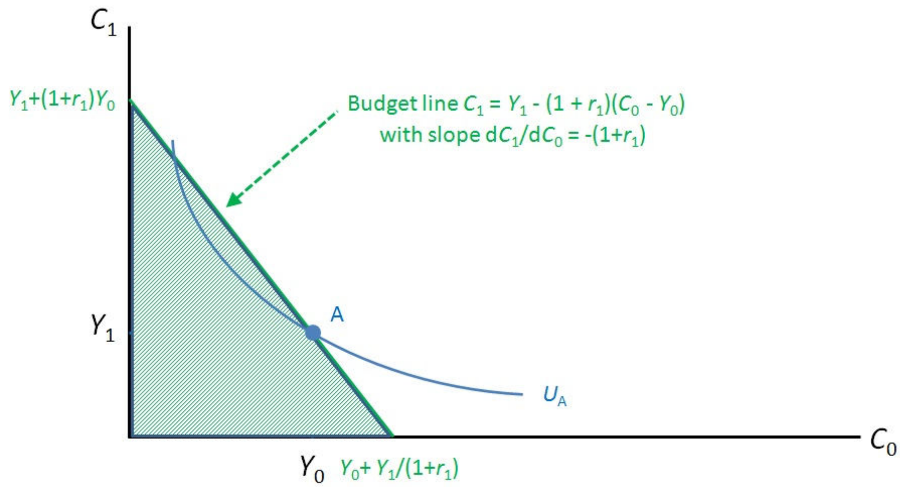
$$0 = Y_1 - (1+r_1)(C_0 - Y_0)$$
$$\rightarrow C_0 = Y_0 + \frac{Y_1}{1+r_1}.$$

- Moreover, the economy's budget line gives:

$$C_1 = Y_1 - (1+r_1)(C_0 - Y_0)$$
$$\rightarrow dC_1 = -(1+r_1)dC_0$$
$$\text{(taking the total derivative; recall that } Y \text{ and } r_1 \text{ are exogenous)}$$
$$\rightarrow \frac{dC_1}{dC_0} = -(1+r_1)$$

which is the budget line's slope.

- Thus, when the economy is open, the following representation of the (open-economy) budget line can emerge (there are two important things to note in the figure below: first, everything on the lower left-hand side bounded above by the budget line falls within the open-economy budget constraint—the hatched triangle; second, the autarky consumption point A will **always** be on the open-economy budget line—if A was a consumption option under autarky, then it **must definitely** be available in the open-economy context as well):

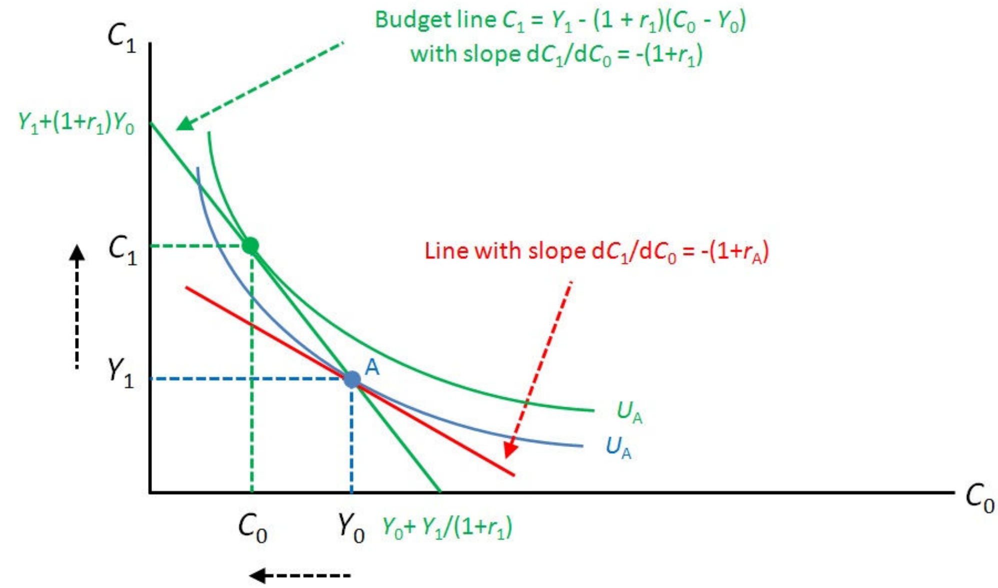


- Note that at the optimal consumption point in the graph above, the indifference curve (with slope $-u'(C_0)/\beta u'(C_1)$) is tangent to the budget line (with slope $-(1 + r_1)$); therefore, at the optimum:

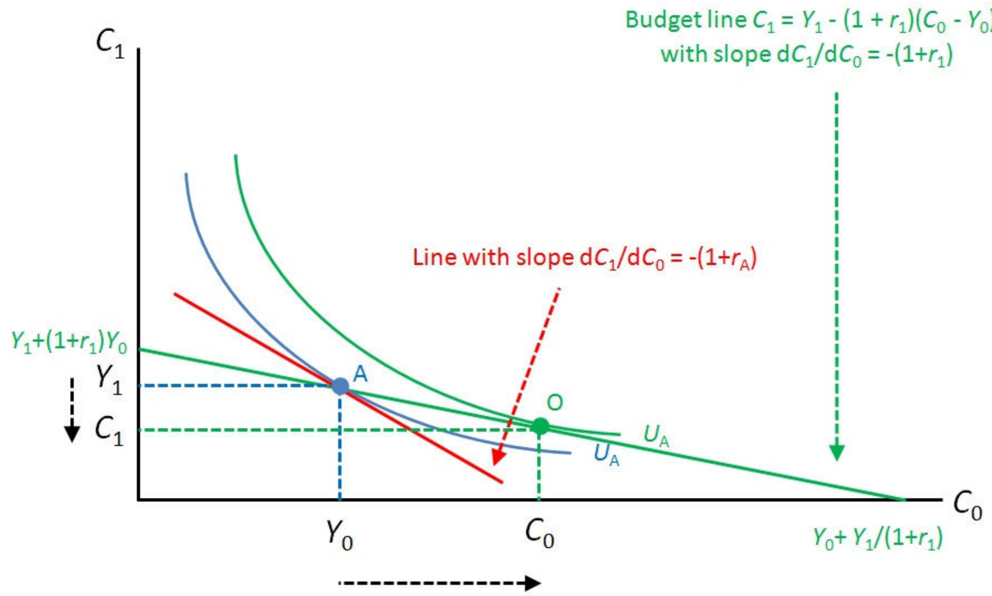
$$-\frac{u'(C_0)}{\beta u'(C_1)} = -(1 + r_1)$$
$$\rightarrow \frac{\beta u'(C_1)}{u'(C_0)} = \frac{1}{1 + r_1},$$

as derived earlier mathematically; this statement is just he Euler equation.

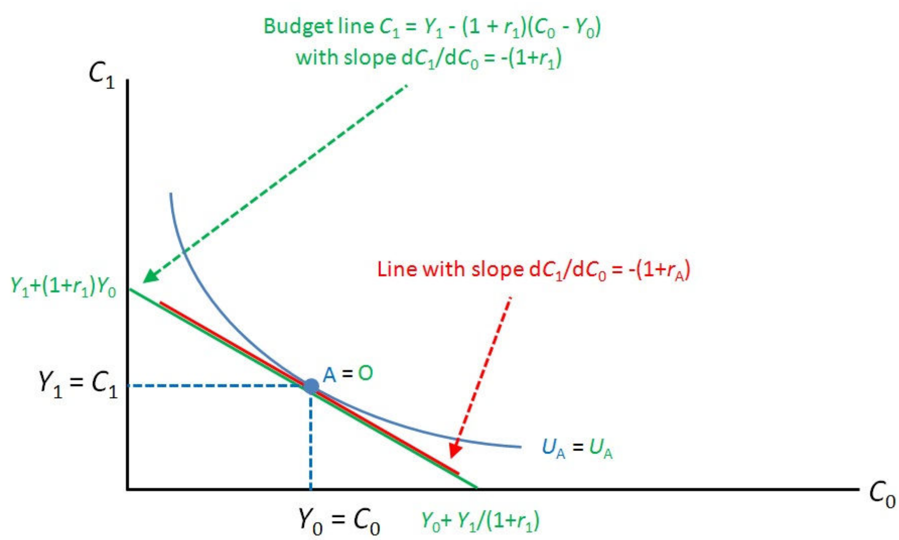
- Now, let's assume that in the open-economy case $r_1 > r^A$. So, the world interest rate is higher than the autarky interest rate. Intuitively, this means that the opportunity cost of consuming in period 0 (which is consumption in period 1) is high. In other words, because the world interest rate is higher than the autarky interest rate, the economy can benefit from lending out some of its endowment in period 0 and being able to consume more in period 1 once it gets repaid with interest. Graphically, as shown below in this case the economy benefits from shifting consumption from the autarky bundle at point A to the open-economy consumption bundle at point O, which puts it on a higher indifference curve (note that $r_1 > r^A$ means that $-(1 + r_1) < -(1 + r_A)$ so the open-economy budget line is steeper than the line tangent to the autarky consumption point referred to earlier):



- Conversely, let's assume that in the open-economy case $r_1 < r^A$. So, the world interest rate is lower than the autarky interest rate. Intuitively, this means that the opportunity cost of consuming in period 0 (which is consumption in period 1) is low. In other words, because the world interest rate is lower than the autarky interest rate, the economy can benefit from borrowing to consume above its endowment in period 0 and paying back with interest in period 1. Graphically, as shown below in this case the economy benefits from shifting consumption from the autarky bundle at point A to the open-economy consumption bundle at point O, which puts it on a higher indifference curve (note that $r_1 < r^A$ means that $-(1 + r_1) > -(1 + r_A)$ so the open-economy budget less steep than the line tangent to the autarky consumption point referred to earlier):



- Finally, let's consider the following case: What happens if $r_1 = r^A$? So, the world interest rate is exactly equal to the autarky interest rate. Intuitively, this means that there are no gains from participating in international financial transactions, so, per the graph below, the economy would simply consume at the autarky consumption point A, which in this case by construction is exactly the same as its open-economy consumption point O.



- **Aside from shedding light on the economy's optimal actions given different world interest rates, note that the preceding analysis implies that in this case of homogeneous agents when the economy is open it can never be worse**

off compared to the case in which it is in autarky.

2.4.3 What does all this mean for the Current Account?

- Recall that, generically, in the open-economy case:

$$Y_t + (1 + r_t)A_t = C_t + A_{t+1};$$

and

$$CA_t = A_{t+1} - A_t.$$

- In the two-period case, $CA_0 = A_1 - A_0 = A_1$ (by assumption no predetermined asset claims when the economy is "born", so $A_0 = 0$), and $CA_1 = A_2 - A_1 = -A_1$ (no claims left over after the economy ends—*transversality condition*; so $A_2 = 0$). Therefore:

$$\begin{gathered} Y_0 = C_0 + A_1 \\ (\text{by } A_0 = 0) \\ \rightarrow Y_0 - C_0 = A_1 \\ \rightarrow Y_0 - C_0 = CA_0 \\ (\text{by } CA_0 = A_1 - A_0 = A_1); \end{gathered}$$

and:

$$\begin{gathered} Y_1 + (1 + r_1)A_1 = C_1 \\ (\text{by } A_2 = 0) \\ \rightarrow Y_1 - C_1 = -(1 + r_1)A_1 \\ \rightarrow Y_1 - C_1 = (1 + r_1)CA_1 \\ (\text{by } CA_1 = A_2 - A_1 = -A_1). \end{gathered}$$

So, if $Y_0 - C_0 < 0$ in the first period the economy consumes more than its output endowment and runs a first-period current account deficit: $CA_0 < 0$. This means that in the second period it must be the case that $Y_1 - C_1 > 0$ and the economy consumes less than its output endowment and runs a second-period current account surplus: $CA_0 > 0$. Conversely, if $Y_0 - C_0 > 0$ in the first period the economy consumes less than its output endowment and runs a first-period current account surplus: $CA_0 > 0$. This means that in the second period it must be the case that $Y_1 - C_1 < 0$ and the economy consumes more than its output endowment and runs a second-period current account deficit: $CA_1 < 0$. In either case, as long as $r_1 \neq r^A$ the economy is on a higher indifference curve when it is open than under autarky.

- **All told, the main implication is that a current account deficit is not necessarily something "bad."**

2.5 Government Consumption

- Let's now move forward and extend the two-period model to include government consumption. We continue to assume that output is exogenous and there's no associated uncertainty with the level of output in any period.

- Government consumption is beyond the private sector's control (it is exogenous to the model), so it is taken as given when the household solves its maximization problem.

- Furthermore, suppose that the government collects T_t in lump-sum taxes every period and runs a balanced budget so that $T_t = G_t\ \forall t$. Therefore the economy's intertemporal budget constraint becomes:

$$
\begin{aligned}
C_0 + \frac{C_1}{1+r_1} &= Y_0 - T_0 + \frac{Y_1 - T_1}{1+r_1} \\
\rightarrow C_0 + \frac{C_1}{1+r_1} &= Y_0 - G_0 + \frac{Y_1 - G_1}{1+r_1}.
\end{aligned}
$$

  So, private consumption now depends on the output endowment that's left over after the government's consumption (and taxation) decision. Because government consumption is implicitly taken from the output endowment (the price of which is normalized to 1), then the price of government consumption is 1. To fix ideas, assume that complete certainty extends to the government's actions, so that G_t, and therefore T_t, are known in advance and with complete certainty $\forall t$.

- In this context, the date t current account becomes:

$$
\begin{aligned}
CA_t &= A_{t+1} - A_t \\
&= Y_t + r_t A_t - C_t - G_t.
\end{aligned}
$$

- *Assuming the same lifetime utility function for the household as in the previous section, it is straightforward to show that because government consumption is exogenous, then the Euler equation is the same as before (you should check this yourself by setting up and solving the household's lifetime utility maximization problem). In fact, notice that from the viewpoint of the household, because both the endowment and government consumption are exogenous, then we could define*

$$
\hat{Y}_t \equiv Y_t - G_t,
$$

  in which case

$$
\begin{aligned}
C_0 + \frac{C_1}{1+r_1} &= Y_0 - G_0 + \frac{Y_1 - G_1}{1+r_1} \\
&= \hat{Y}_0 + \frac{\hat{Y}_1}{1+r_1},
\end{aligned}
$$

  which makes clear that all that's going on is that for any period in which government *consumption is positive then there's just less output endowment around for private consumption.*

- **Suppose for all that follows under this government consumption section that $\beta = 1/(1+r_1)$, which is the case in which consumption smoothing is desirable. Furthermore, assume that $Y_0 = Y_1 = Y$.**

- One case to consider, while admittedly boring, is the context in which $G_0 = G_1 = 0$. In this case it is straightforward that $C_0 = C_1 = Y$ ($r^A = r_1$), which means that $CA_0 = CA_1 = 0$: the current account is balanced through time (and the economy optimally consumes its autarky bundle).

- The next case to examine, which is more interesting, is the one in which $G_0 > 0$ and $G_1 = 0$, then $Y_0 = Y - G_0 < Y_1 = Y$.

  - Of course, it is still the case that the household wants to attain the same consumption in each period (recall that we have $\beta = 1/(1+r_1)$), so the household desires $C_0 = C_1 = C$. In order to smooth consumption, the household will borrow in period 0 against its relatively high period-1 after-tax income to shift part of the burden of the temporary taxes to the future. So, there will be a current account deficit in the first period and a current account surplus in the second period.

  - As before, we can solve for the desired level of constant consumption $C_0 = C_1 = C$ using the intertemporal budget constraint:

$$C_0 + \frac{C_1}{1+r_1} = Y_0 - G_0 + \frac{Y_1 - G_1}{1+r_1}$$
$$\rightarrow C_0 + \frac{C_1}{1+r_1} = Y_0 - G_0 + \frac{Y_1}{1+r_1}$$
$$\text{(recall } G_1 = 0 \text{ by assumption)}$$
$$\rightarrow C + \frac{C}{1+r_1} = Y - G_0 + \frac{Y}{1+r_1}$$
$$\text{(because } Y_0 = Y_1 = Y\text{)}$$
$$\rightarrow (1+r_1)\,C + C = (1+r_1)\,(Y - G_0) + Y$$
$$\rightarrow (2+r_1)C = (1+r_1)\,(Y - G_0) + Y$$
$$\rightarrow C = \frac{(1+r_1)\,(Y - G_0) + Y}{2+r_1}.$$

  Rearranging:

$$C = Y - \frac{1+r_1}{2+r_1} G_0.$$

  This means that government consumption in period 0 lowers private consumption <u>each</u> period by an amount smaller than G_0 ($(1+r_1)\,/\,(2+r_1) < 1$). But this result is only because taxes are temporary and government consumption occurs in the first period only. In essence, financial openness allows the household to distribute the tax burden across time.

- Another case to examine is the one in which $G_0 = G_1 = G > 0$.

  - Then,

$$
\begin{gathered}
C_0 + \frac{C_1}{1+r_1} = Y_0 - G_0 + \frac{Y_1 - G_1}{1+r_1} \\
\rightarrow C + \frac{C}{1+r_1} = Y - G + \frac{Y-G}{1+r_1} \\
\rightarrow (1+r_1)\, C + C = (1+r_1)\,(Y-G) + Y - G \\
\rightarrow (2+r_1)C = (1+r_1)\,(Y_0 - G) + Y - G \\
\rightarrow C = \frac{(1+r_1)\,(Y-G) + Y - G}{2+r_1}. \\
\rightarrow C = Y - G.
\end{gathered}
$$

- Recall that in the current development:

$$CA_t = A_{t+1} - A_t.$$

Therefore, $CA_0 = A_1 - A_0 = A_1$ (we continue to assume that there are no predetermined asset claims when the economy is "born"). And, $CA_1 = A_2 - A_1 = -A_1$ (by the transversality condition, no claims are optimally left over after the economy ends). The budget constraint is now, generically:

$$Y_t - G_t + (1+r_t)A_t = C_t + A_{t+1}.$$

Therefore:

$$
\begin{gathered}
Y_0 = C_0 + G_0 + A_1 \\
(\text{by } A_0 = 0) \\
\rightarrow Y_0 - C_0 - G_0 = A_1 \\
\rightarrow Y_0 - C_0 - G_0 = CA_0 \\
(\text{by } CA_0 = A_1 - A_0 = A_1);
\end{gathered}
$$

and

$$
\begin{gathered}
Y_1 - G_1 + (1+r_1)A_1 = C_1 \\
(\text{by } A_2 = 0) \\
\rightarrow Y_1 - C_1 - G_1 = -(1+r_1)A_1 \\
\rightarrow Y_1 - C_1 - G_1 = (1+r_1)CA_1 \\
(\text{by } CA_1 = A_2 - A_1 = -A_1).
\end{gathered}
$$

  - In the first case we examined in which $Y_0 = Y_1 = Y$ and $G_0 > 0$ and $G_1 = 0$, because $CA_0 = -CA_1$:

$$\begin{aligned} -CA_1 &= Y - C - G_0 \\ &= Y - \underbrace{Y + \frac{1+r_1}{2+r_1}G_0}_{=-C} - G_0 \\ &= \frac{1+r_1-2-r_1}{2+r_1}G_0 \\ \rightarrow CA_1 &= -\frac{G_0}{2+r_1} < 0. \end{aligned}$$

- In the second case we examined in which $Y_0 = Y_1 = Y$ and $G_0 = G_1 = G > 0$, because $CA_0 = -CA_1$:

$$\begin{aligned} -CA_1 &= Y - \bar{C} - G \\ &= Y - \underbrace{Y + G}_{=-\bar{C}} - G = 0. \end{aligned}$$

- **So, government consumption affects the current account only to the extent that it tilts the relative paths of private <u>net</u> income. If the paths are affected equally, then the current account is balanced across periods.**

2.6 Investment

- We extend the two-period model developed so far to include private investment investment. Unless otherwise noted, all assumptions stated so far in these lecture notes carry over. **Adding investment implies that we can now shift over to a world in which production is endogenous.** In particular, let:

$$Y_t = F(K_t),$$

that is, a period's output, Y_t, is a function F of that period's capital, K_t. Assume $F'(K_t) > 0$ and $F''(K_t) < 0$. So, output is increasing in capital, but subject to diminishing marginal productivity. Of course, output cannot be produced without capital so we assume that $F(0) = 0$.

  - **For simplicity we assume that all of the domestic capital stock is owned by domestic residents, and we continue to abstract from labor.**
  - There is no stochastic productivity shock as in the RBC frameowork, so it continues to be the case that there is no need for expectation operators.

- The representative household owns the production technology. Capital takes time to build, and the capital accumulation equation is

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where: $\delta \in (0, 1)$ is the constant depreciation rate (δ is the Greek letter "delta") and I_t is (private) investment. As in the RBC model, this equation implies that today's capital stock is predetermined: it depends on the capital stock that was there yesterday and the fraction of the capital stock that remains today net of depreciation, as well as yesterday's investment.

- Output Y_t can be used either for consumption (private or government) or investment. Because we continue to assume, as before, that the price of consumption (and hence the price of output) is normalized to 1, then the price of investment and capital is also 1. If output is invested, then next period it becomes capital and is only then ready for use as such. The process of creating capital is **reversible** in the sense that existing capital can be "eaten" (transformed into private or government consumption). Indeed, note from the capital accumulation equation that nothing restricts investment from being negative (negative investment is the way through which the economy can "eat" part or all of its available capital stock—just think of this case as being one in which the capital stock is sold off and the proceeds are consumed).

- **Domestic private wealth** at is given by $A_t + K_t$, that is, the sum of net foreign assets and the domestic capital stock.

- The economy's budget constraint is:

$$
\begin{gathered}
Y_t + r_t A_t = (C_t + I_t + G_t) + CA_t \\
\rightarrow I_t = (Y_t + r_t A_t - C_t - G_t) - CA_t
\end{gathered}
$$

  Define <u>*savings*</u>: $S_t \equiv Y_t + r_t A_t - C_t - G_t$. Therefore:

$$
S_t - I_t = CA_t.
$$

- We can rearrange the economy's budget constraint as follows:

$$
\begin{gathered}
Y_t + r_t A_t - C_t - I_t - G_t = A_{t+1} - A_t \\
\text{(because } CA_t = A_{t+1} - A_t\text{)} \\
\rightarrow Y_t + (1 + r_t) A_t = C_t + I_t + G_t + A_{t+1},
\end{gathered}
$$

  which using the capital accumulation to substitute out investment yields:

$$
Y_t + (1 + r_t) A_t = C_t + \underbrace{K_{t+1} - (1 - \delta) K_t}_{=I_t} + G_t + A_{t+1}.
$$

- We continue to assume that the economy is "born" in period 0 and that that $A_0 = 0$; the economy lives through period 1, and optimality requires that $A_2 = 0$ (transversality condition). **In addition, we assume that when the economy is born it is endowed with capital stock K_0. But, the subsequent capital stock evolves according to the capital accumulation equation from before. In essence, this means that $Y_0 = F(K_0)$ is given.**

- Because:

$$\begin{aligned} Y_t + r_t A_t - C_t - I_t - G_t &= CA_t \\ &= A_{t+1} - A_t, \end{aligned}$$

then in period 0:

$$\begin{gathered} Y_0 + r_0 A_0 - C_0 - I_0 - G_0 = CA_0 \\ \rightarrow Y_0 - C_0 - I_0 - G_0 = A_1 \\ \text{(given } A_0 = 0 \text{ by assumption)}, \end{gathered}$$

and in period 1:

$$\begin{gathered} Y_1 + r_1 A_1 - C_1 - I_1 - G_1 = CA_1 \\ \rightarrow Y_1 + r_1 A_1 - C_1 - I_1 - G_1 = -A_1 \\ \text{(by } A_2 = 0 \text{ optimally)}. \end{gathered}$$

The period-1 constraint implies that:

$$\begin{gathered} (1 + r_1) A_1 = -Y_1 + C_1 + I_1 + G_1 \\ \rightarrow A_1 = \frac{-Y_1 + C_1 + I_1 + G_1}{1 + r_1}. \end{gathered}$$

Substitute this last expression in the first period constraint:

$$\begin{gathered} Y_0 - C_0 - I_0 - G_0 = A_1 \\ \rightarrow Y_0 - C_0 - I_0 - G_0 = \frac{-Y_1 + C_1 + I_1 + G_1}{1 + r_1} \\ \rightarrow C_0 + I_0 + \frac{C_1 + I_1}{1 + r_1} = Y_0 - G_0 + \frac{Y_1 - G_1}{1 + r_1}, \end{gathered}$$

which again is the intertemporal budget constraint.

  - **Now, this means that, given investment, it is the present value of consumption <u>plus</u> investment that is limited by the present value of output <u>net</u> of exogenous government expenditures.**

- By the way, we're heading towards solving the economy's maximization problem in this context, which involves some rearrangement and first-order conditions that I know you all can't get enough of! It's because I know you love doing this so much that I keep on setting up all this sort of analysis ;) Just kidding, I know a lot of this is a pain, but if you look back hopefully by now you feel like you've learned a ton of interesting stuff!

- Moving along, because output is endogenous, the preceding equation can be restated as:

$$C_0 + I_0 + \frac{C_1 + I_1}{1 + r_1} = F(K_0) - G_0 + \frac{F(K_1) - G_1}{1 + r_1}.$$

Using the fact that

$$I_t = K_{t+1} - (1-\delta)K_t,$$

we can restate the intertemporal budget constraint as:

$$C_0 + \underbrace{K_1 - (1-\delta)K_0}_{=I_0} + \frac{C_1 + \overbrace{K_2 - (1-\delta)K_1}^{=I_1}}{1+r_1} = F(K_0) - G_0 + \frac{F(K_1) - G_1}{1+r_1}.$$

- Because there is no period 2, then there is no point in leaving any uneaten capital at the end of period 1, which means that $K_2 = 0$ (another transversality condition!). Note that since $K_2 = (1-\delta)K_1 + I_1$, $K_2 = 0$ means that $I_1 = -(1-\delta)K_1$: disinvestment! In particular, this means that the household uses K_1, the capital available **at the beginning** of period 1, to produce $F(K_1)$ and once production has taken place and capital has depreciated, the household eats the remaining capital stock, which consists of $(1-\delta)K_1$, right before the economy disappears. Implement $K_2 = 0$ in the intertemporal budget constraint and solve for C_1:

$$\begin{aligned} & C_0 + K_1 - (1-\delta)K_0 + \frac{C_1 - (1-\delta)K_1}{1+r_1} = F(K_0) - G_0 + \frac{F(K_1) - G_1}{1+r_1} \\ \to\ & (1+r_1)\left[C_0 + K_1 - (1-\delta)K_0\right] + C_1 - (1-\delta)K_1 = (1+r_1)\left[F(K_0) - G_0\right] + F(K_1) - G_1 \\ \to\ & C_1 = \underbrace{(1-\delta)K_1}_{=-I_1} + F(K_1) - G_1 - (1+r_1)[C_0 + \underbrace{K_1 - (1-\delta)K_0}_{=I_0} - F(K_0) + G_0]. \end{aligned} \tag{2}$$

- All told, the economy's problem can be stated as:

$$\begin{aligned} \max_{C_t,\ K_{t+1}} U_0 &= \sum\nolimits_{t=0}^{1} \beta^t u(C_t) \\ &= \beta^0 u(C_0) + \beta^1 u(C_1) \\ &= u(C_0) + \beta u(C_1) \\ &= \max_{C_0,\ K_1} u(C_0) + \beta \cdot \underbrace{u(C_1(C_0, K_1 . \cdot))}_{C_1 \text{ is a function of the choice variables } C_0 \text{ and } K_1 \text{ and other things}} \end{aligned}$$

such that equation (2) holds (because today's capital stock is predetermined from yesterday's choices—and in the case of K_0 simply by endowment—then it is tomorrow's capital stock that is in fact a choice variable). Substitute the constraint into the objective function to yield:

$$\begin{aligned} \max_{C_0,\ K_1} U_0 = u(C_0) + \beta u((1-\delta)K_1 + F(K_1) - G_1 \\ -(1+r_1)\left[C_0 + K_1 - (1-\delta)K_0 - F(K_0) + G_0\right]). \end{aligned}$$

The first-order condition for C_0 is:

$$\frac{\partial U_0}{\partial C_0} \stackrel{!}{=} 0$$
$$\rightarrow u'(C_0) - \beta u'(C_1)(1+r_1) = 0$$
$$\text{(using the chain rule)}$$
$$\rightarrow \frac{1}{1+r_1} = \frac{\beta u'(C_1)}{u'(C_0)},$$

which is the Euler equation as usual. The first-order condition for K_1 is:

$$\frac{\partial U_0}{\partial K_1} \stackrel{!}{=} 0$$
$$\rightarrow \beta u'(C_1) \cdot [(1-\delta) + F'(K_1) - (1+r_1)] \stackrel{!}{=} 0.$$

Because $C_1 > 0$, then $u'(C_1) > 0$ so we can divide by $\beta u'(C_1)$ and the previous equation can be stated as:

$$(1-\delta) + F'(K_1) - (1+r_1) = 0$$
$$\rightarrow F'(K_1) = r_1 + \delta.$$

- This equation says that the choice of capital in period 1 is such that capital's period-1 marginal return is the same as that on a foreign loan plus the depreciation rate. Alternatively, this equation can be interpreted as saying that investment in period 0 should continue until the marginal return on this investment is equal to the return on a foreign loan plus depreciation. **An important point: in this small open economy in which the household faces a perfect world capital market, government consumption does not crowd out (private) investment.**

3 Open Economy Real Business Cycle Models

- We now proceed to develop a benchmark international RBC model. We no longer assume a small open economy context. Instead, our framework now focuses on two **large** open economies (e.g., the United States vs. the rest of the world). Each country is inhabited by a continuum of infinitely lived identical individuals grouped into an aggregate risk sharing household. Production in each country takes place through a representative final goods producing firm. All non-price variables are normalized by the **world** population, which consists of a unit mass. The **exogenously determined** and constant fraction of the world population in the home country is ψ (the Greek letter "psi") and, therefore, the fraction of the population in the foreign country is $1-\psi$. All price variables are normalized by the price of consumption, which itself is therefore normalized to 1. The final consumption good, **which is identical across the world and produced by both economies**, is traded internationally, and we assume that international financial markets are such that they allow complete risk sharing across economies.

- We focus on the planning solution (instead of the decentralized economy) so we can focus on the most relevant model implications and omit direct reference to prices. Just like in the closed economy RBC model, though, we could instead solve a decentralized problem where prices appear explicitly Recall that as mentioned earlier, in a large open-economy context the world interest is endogenous from the viewpoint of each economy as they recognize that they are large players in international financial markets.

- For simplicity, we also omit reference to capital (and therefore investment) as well as government spending; thus, we assume that production only uses labor as an input. Country-specific production is subject to idiosyncratic exogenous stochastic productivity process, so we need to make use of expectations operators.

3.1 Utility and Production

- Consider two large open economies, home and foreign (foreign variables that need not be equal to home variables are denoted by an asterisk). Each economy's lifetime utility is given by:

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t) - h(N_t) \right\} \text{ and } U^* = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t^*) - h\left(N_t^*\right) \right\},$$

where: $\mathbb{E}_t$ is the expectation operator; $\beta \in (0,1)$ is the (constant) subjective discount factor (β is the Greek letter "beta"; C and C^* denote consumption; N and N^* denote labor; $u' > 0$, $u'' < 0$, $h' > 0$, and $h'' > 0$. Production of the final good in each country is given by:

$$Y_t = Z_t N_t^{\alpha} \text{ and } Y_t^* = Z_t^* \left(N_t^*\right)^{\alpha},$$

where: Z and Z^* are stochastic technology processes and $\alpha \in (0,1)$.

3.2 Exogenous Productivity and International Linkages

- Productivity processes capture the possibility of international productivity spillovers. This processes can be stated in VAR format as:

$$\begin{bmatrix} \ln Z_t \\ \ln Z_t^* \end{bmatrix} = \begin{bmatrix} \rho & \upsilon^* \\ \upsilon & \rho^* \end{bmatrix} \begin{bmatrix} \ln Z_{t-1} \\ \ln Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix},$$

where; $\rho, \rho^* > 0$ (ρ is the Greek letter "rho"); $\upsilon, \upsilon^* > 0$ (υ is the Greek letter "upsilon"); $\mathbb{E}_t\left(\varepsilon_t\right) = \mathbb{E}_t\left(\varepsilon_t^*\right) = 0$; and the standard deviation of ε and ε^* is denoted, respectively, by σ_ε and σ_{ε^*} (ε is the Greek letter "epsilon" and σ is the Greek letter "sigma"). Under this specification, innovations to productivity that originate in one country (ε_t or ε_t^*) are transmitted to the other country via the "diffusion" parameters, υ and υ^*, with a 1-period lag. The "persistence" parameters, ρ and ρ^*, are important for the serial correlation of the productivity variable within a country. The variance-covariance matrix for the innovations to the productivity process is assumed to be symmetric (although it need not be) so that:

$$\mathbb{E}_t\left(\varepsilon_t, \varepsilon_t^*\right)\left(\varepsilon_t, \varepsilon_t^*\right)' = \begin{bmatrix} \sigma_\varepsilon^2 & \kappa \\ \kappa & \sigma_\varepsilon^2 \end{bmatrix},$$

where: κ is the Greek letter "kappa."

- Note that using matrix multiplication, the system of productivity processes can be stated as

$$\ln Z_t = \rho \ln Z_{t-1} + v^* \ln Z_{t-1}^* + \varepsilon_t$$

and

$$\ln Z_t^* = \rho^* \ln Z_{t-1}^* + v \ln Z_{t-1} + \varepsilon_t^*.$$

3.3 Planning Problem

- A benevolent world social planner solves the following problem:

$$\max_{C_t, C_t^*, N_t, N_t^*} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{\psi\left[u(C_t) - h(N_t)\right] + (1-\psi)\left[u(C_t^*) - h\left(N_t^*\right)\right]\},$$

such that:

$$\begin{aligned} C_t + NX_t &\leq Y_t, \\ C_t^* + NX_t^* &\leq Y_t^*, \end{aligned}$$

$$Y_t = Z_t N_t^\alpha, \tag{3}$$

$$Y_t^* = Z_t^*\left(N_t^*\right)^\alpha, \tag{4}$$

where: $0 < \alpha < 1$ (α is the Greek letter "alpha"); and NX and NX^* denote net exports for the home and foreign economy, respectively. Note that the planner's setup uses as Pareto weights the fraction of the population in each country.

- **Define** net exports in the home and foreign country, respectively, as:

$$NX_t = Y_t - C_t \tag{5}$$

and

$$NX_t^* = Y_t^* - C_t^*. \tag{6}$$

Of course, because this is a 2-economy world, then whatever one economy exports the other imports. As such, $NX_t = -NX_t^*$, so we can combine all of the preceding constraints into a single aggregate resource constraint:

$$C_t + C_t^* \leq Y_t - NX_t + Y_t^* - NX_t^*$$
$$\rightarrow C_t + C_t^* \leq Z_t N_t^\alpha + Z_t^* \left(N_t^*\right)^\alpha .$$

- The benevolent world social planner's current value Lagrangian is:

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{\psi \left[u(C_t) - h(N_t)\right] + (1-\psi)\left[u(C_t^*) - h\left(N_t^*\right)\right]$$
$$+\lambda_t [Z_t N_t^\alpha + Z_t^* \left(N_t^*\right)^\alpha - C_t - C_t^*]\},$$

and the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t} \stackrel{!}{=} 0$$
$$\rightarrow \psi u'(C_t) - \lambda_t \stackrel{!}{=} 0;$$

$$\frac{\partial \mathcal{L}}{\partial C_t^*} \stackrel{!}{=} 0$$
$$\rightarrow (1-\psi)\, u'(C_t^*) - \lambda_t \stackrel{!}{=} 0;$$

$$\frac{\partial \mathcal{L}}{\partial N_t} \stackrel{!}{=} 0$$
$$\rightarrow -\psi h'(N_t) + \lambda_t \alpha Z_t N_t^{\alpha-1} = 0;$$

$$\frac{\partial \mathcal{L}}{\partial N_t^*} \stackrel{!}{=} 0$$
$$\rightarrow -(1-\psi) h'\left(N_t^*\right) + \lambda_t \alpha Z_t^* \left(N_t^*\right)^{\alpha-1} = 0;$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} \stackrel{!}{=} 0$$
$$\rightarrow C_t + C_t^* \leq Z_t N_t^\alpha + Z_t^* \left(N_t^*\right)^\alpha . \tag{7}$$

- Combine the first two first order conditions to yield the "consumption sharing condition"

$$\psi u'(C_t) = (1-\psi)\, u'(C_t^*). \tag{8}$$

Thus, given trade in final goods and complete international risk sharing the planning outcome implies that the **level** consumption across countries should only differ to the extent that the Pareto weights are different. Moreover, the consumption sharing condition implies that

$$\begin{aligned}
cov\left[\psi u'(C_t),(1-\psi)\,u'(C_t^*)\right] &= \mathbb{E}\left\{\begin{array}{c}\{\psi u'(C_t)-\mathbb{E}\left[\psi u'(C_t)\right]\}\\ \cdot\{(1-\psi)\,u'(C_t^*)-\mathbb{E}\left[(1-\psi)\,u'(C_t^*)\right]\}\end{array}\right\}\\
&= \mathbb{E}\left\{\{\psi u'(C_t)-\mathbb{E}\left[\psi u'(C_t)\right]\}\cdot\{\psi u'(C_t)-\mathbb{E}\left[\psi u'(C_t)\right]\}\right\}\\
&\quad \text{(by the consumption sharing condition)}\\
&= \mathbb{E}\left\{\{\psi u'(C_t)-\mathbb{E}\left[\psi u'(C_t)\right]\}^2\right\}\\
&= var\left[\psi u'(C_t)\right].
\end{aligned}$$

In addition, the standard deviation of $\psi u'(C_t)$ is

$$\sqrt{var\left[\psi u'(C_t)\right]}$$

and the standard deviation of $(1-\psi)\,u'(C_t^*)$ is

$$\sqrt{var\left[(1-\psi)\,u'(C_t^*)\right]}.$$

So,

$$\begin{aligned}
corr\left[\psi u'(C_t),(1-\psi)\,u'(C_t^*)\right] &= \frac{cov\left[\psi u'(C_t),(1-\psi)\,u'(C_t^*)\right]}{\sqrt{var\left[\psi u'(C_t)\right]}\cdot\sqrt{var\left[(1-\psi)\,u'(C_t^*)\right]}}\\
&= \frac{var\left[\psi u'(C_t)\right]}{\sqrt{var\left[\psi u'(C_t)\right]}\cdot\sqrt{var\left[(1-\psi)\,u'(C_t^*)\right]}}\\
&= \frac{var\left[\psi u'(C_t)\right]}{\sqrt{var\left[\psi u'(C_t)\right]}\cdot\sqrt{var\left[\psi u'(C_t)\right]}}\\
&\quad \text{(by the consumption sharing condition)}\\
&= \frac{var\left[\psi u'(C_t)\right]}{var\left[\psi u'(C_t)\right]}\\
&= 1.
\end{aligned}$$

Therefore, the consumption sharing condition implies that optimally consumption should be perfectly correlated across countries. This means that the planner wants consumption risk completely gotten rid of even if the level of consumption differs across countries in any given period.

- Also, taking logs of the consumption sharing condition we have:

$$\begin{gathered}
\psi u'(C_t) = (1-\psi)\,u'(C_t^*)\\
\to \ln\psi u'(C_t) = \ln(1-\psi)\,u'(C_t^*)\\
\to \ln\psi + \ln u'(C_t) = \ln(1-\psi) + \ln u'(C_t^*),
\end{gathered}$$

and total differentiation, implies that:

$$\begin{gathered} d\ln\psi + d\ln u'(C_t) = d\ln(1-\psi) + d\ln u'(C_t^*) \\ \to 0 + d\ln u'(C_t) = 0 + d\ln u'(C_t^*) \\ \text{(since } \psi \text{ is assumed to be constant)} \\ \to d\ln u'(C_t) = d\ln u'(C_t^*). \end{gathered}$$

So, the growth rate of marginal instantaneous consumption utility is equalized across countries. Moreover, if $u(C_t) = \ln(C_t)$ and $u(C_t^*) = \ln(C_t^*)$ then $u'(C_t) = 1/C_t$ and $u'(C_t^*) = 1/C_t^*$, which means that the preceding equation can be restated:

$$\begin{gathered} d\ln u'(C_t) = d\ln u'(C_t^*) \\ \to d\ln\left(\frac{1}{C_t}\right) = d\ln\left(\frac{1}{C_t^*}\right) \\ \to d\ln(1) - d\ln C_t = d\ln(1) - d\ln C_t^* \\ \to 0 - d\ln C_t = 0 - d\ln C_t^* \\ \to d\ln C_t = d\ln C_t^*, \end{gathered}$$

Thus, under log utility the consumption sharing condition implies that the growth rate of consumption should be equalized across countries.

- Finally, turning to the first-order conditions for labor, combine these with on a country-specific basis with the corresponding first-order conditions for consumption to obtain:

$$\begin{gathered} \psi h'(N_t) = \lambda_t \alpha Z_t N_t^{\alpha-1} \\ \to \psi h'(N_t) = \psi u'(C_t)\alpha Z_t N_t^{\alpha-1} \\ \to h'(N_t) = u'(C_t)\alpha Z_t N_t^{\alpha-1}; \end{gathered} \tag{9}$$

and, analogously,

$$h'\left(N_t^*\right) = \lambda_t \alpha Z_t^* \left(N_t^*\right)^{\alpha-1}. \tag{10}$$

- The bottom line is that the solution to the planning problem involves a system of 8 unknowns, C, C^*, N, N^*, Y, Y^*, NX, NX^*, in the 8 preceding numbered equations corresponding to this section.

3.4 Advanced Extensions

- A large chunk of what we've done in this class has been building up to the International RBC model we just developed. This model is the workhorse of modern International Macroeconomics for real business cycles, and, with extensions, for academic analysis of monetary, fiscal, and macroprudential policy as well. In that same vein, there are countless extensions of this model to deal with a very wide range of topics. Because of its workhorse status, after wrapping up this Lesson you should be quite comfortable dealing with modern International Macroeconomics in whatever scenario you may encounter it in the future.

- That said, bear in mind that the workhorse model is a benchmark akin to what perfect competition is in Microeconomics. The assumption of perfect competition has limitations, hence all of the extensions of this model. Similarly, the workhorse model has many limitations, and many of these limitations (deemed "puzzles") are at the frontier of current International Macroeconomics research.

- A classic reference for many of these puzzles is: Obstfeld, Maurice, and Kenneth Rogoff. 2000. "The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?." *NBER Macroeconomics Annual*, 15: pp. 339-390. Yup, this paper is nearly 20 years old, but, again it is a classic reference. One of the major puzzles involves consumption correlation across countries. Recall that in Lesson 5 we generated we explored a table with stylized facts pertaining to empirical real business cycles. A key stylized fact that emerges from this analysis is that (private) consumption tends to be less correlated across countries than output. *Yet, the workhorse model predicts that consumption should be perfectly correlated across countries, meaning that an implication of the workhorse model is that output across countries should be less correlated than consumption.* There are many explanations for this puzzle, but it is still a hot topic.

- The International Business Cycle literature began with Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. 1992. "International real business cycles." *Journal of political Economy*, 100 (4): pp. 745-775.

- If you are interested in developing your International Macroeconomic skills further, the two papers noted above (and all papers thereafter that cite them) are an excellent starting point. In spite of there being so many interesting things in the literature in addition to what we've covered thus far in the class, given time constraints we are unable to push beyond where we've arrived as related to international real business cycles. However, as mentioned above, the class is structured in such a way that you should easily be able to grasp any advanced extensions you may encounter in the future.

- All told, let me end this lesson by noting the following. In 1924 John Maynard Keynes wrote an obituary essay for economist Alfred Marshall.[1] At the beginning of this essay, Keynes deliberates on the skills needed by an economist. In particular, he notes that:

  > "The study of economics does not seem to require any specialized gifts of an unusually high order. Is it not, intellectually regarded, a very easy subject compared with the higher branches of philosophy or pure science? An easy subject at which few excel! The paradox finds its explanation, perhaps, in that the master-economist must possess a rare combination of gifts. He must be mathematician, historian, statesman, philosopher—in some degree. [**NB: I would add nowadays statistician and coding expert.**] He must understand symbols and speak in words. He must contemplate the particular in terms of the

[1] Keynes, John M. 1924. "Alfred Marshall, 1842-1924." *The Economic Journal*, 34 (135): pp. 311-372.

general and touch abstract and concrete in the same flight of thought. He must study the present in the light of the past for the purposes of the future. No part of man's nature or his institutions must lie entirely outside his regard. He must be purposeful and disinterested in a simultaneous mood; as aloof and incorruptible as an artist, yet sometimes as near to earth as a politician."

- In essence, what we've done in the class is go over a bunch of topics, including math, statistics, and computer programming, all of which are essential in the toolkit of a professional in the discipline of International Macroeconomics. So, we've built up your knowledge in the spirit of what Keynes would have thought was needed for someone to be a professional in this disipline! :)