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# Should a Government Fiscally Intervene in a Recession and, If So, How? 

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#### Abstract

The validity of discretionary fiscal policy in a recession will differ according to the cause and mechanism of recession. In this paper, discretionary fiscal policy in a recession caused by a fundamental shock that changes the steady state downwards is examined. In such a recession, households need to discontinuously increase consumption to a point on the saddle path to maintain Pareto efficiency. However, they will not "jump" consumption in this manner and instead will choose a "Nash equilibrium of a Pareto inefficient path" because they dislike unsmooth and discontinuous consumption and behave strategically. The paper concludes that increasing government consumption until demand meets the present level of production and maintaining this fiscal policy for a long period is the best option. Consequent government debts can be sustainable even if they become extremely large.


JEL Classification code: E20, E32, E62, H20, H30, H63
Keywords: Discretionary Fiscal policy; Recession; Government consumption; Government debts; Pareto inefficiency; Time preference

[^0]
## 1 INTRODUCTION

Discretionary fiscal policy has been studied from many perspectives since the era of Keynes (e.g., Keynes, 1936; Kopcke et al., 2006; Chari et al., 2009; Farmer, 2009; Alesina, 2012; Benhabib et al., 2014). An important issue is whether a government should intervene fiscally in a recession, and if so, how. The answer will differ according to the cause and mechanism of recession. Particularly, it will be different depending on whether "disequilibrium" is generated. The concept of disequilibrium is, however, controversial and therefore arguments continue even now about the use of discretionary fiscal policy in a recession. In this paper, the concept of disequilibrium is not used, but instead the concept of a "Nash equilibrium of a Pareto inefficient path" is used.

Recessions are generated by various shocks (e.g., Rebelo, 2005; Blanchard, 2009; Ireland, 2011; Schmitt-Grohé and Uribe, 2012; McGrattan and Prescott, 2014; Hall, 2016). Some fundamental shocks will change the steady state, and if the steady state is changed downwards (i.e., to lower levels of production and consumption), households must change the consumption path to one that diminishes gradually to the posterior steady state. Therefore, growth rates become negative; that is, a recession begins. However, the explanation of the mechanism of this type of recession is not perfect because an important question still needs to be answered. If households discontinuously increase ("jump up") their consumption from the prior steady state to a point on the posterior saddle path and then gradually move to the posterior steady state, Pareto efficiency is held and thereby unemployment rates do not rise. Therefore, even in a serious and large-scale recession, unemployment does not increase. This is a very unnatural outcome of a serious recession.

Harashima (2004, 2009, 2013a) showed a mechanism by which households do not jump up their consumption even if the steady state is changed downward because they are intrinsically risk averse and non-cooperative and want to smooth consumption. The consumption jump does not give them the highest expected utility; that is, unsmooth and discontinuous consumption is not optimal for households. Hence, instead of choosing the posterior saddle path, they will choose a "Nash equilibrium of a Pareto inefficient path" as the optimal consumption path. Because of its Pareto inefficiency, unemployment rates will increase sharply and stay high during a recession. This paper examines whether discretionary fiscal policy is necessary, and if it is necessary, how it should be implemented when an economy is in a recession and proceeding on such a Pareto inefficient path.

Fundamental shocks that change the steady state basically mean shocks on deep parameters. A representative fundamental shock, an upward shock on the rate of time preference (RTP), is examined in this paper. Faced with this shock, a government has three options: (1) do not intervene, (2) increase government consumption, and (3) cut taxes. The consequences of these options are examined and the outcomes are evaluated to determine which is the best option. I conclude that increasing government consumption until the demand meets the present level of production and maintaining this fiscal policy during the recession is the best option. Nevertheless, this option will be accompanied by large and accumulating government debts, but these debts can be sustained if the government properly increases taxes in the future. This option means that huge government debts will play an essential role as a buffer against negative effects of the fundamental shock.

## 2 A MECHANISM OF RECESSION

### 2.1 An upward RTP shock

There are various possible sources of recession, but in this paper, a recession caused by a fundamental shock, particularly by an upward shift of RTP, is examined because an upward shift of RTP seems to be most likely the cause of the Great Recession (Harashima, 2016). A
technology shock was probably not the cause of the Great Recession because technology does not suddenly and greatly regress. Frictions on price adjustments are also unlikely to be the cause because the micro-foundation of friction does not seem to be sufficiently persuasive (e.g., Mankiw, 2001), particularly the micro-foundation of its persistence. On the other hand, Harashima (2016) showed that an upward RTP shock could explain the occurrence of the Great Recession and showed evidence that the estimated RTP of the United States increased in about 2008.

RTP plays an essential role in economic activities, and its importance has been emphasized since the era of Irving Fisher (Fisher, 1930). One of the most important equations in economics is the steady state condition

$$
\theta=r
$$

where $\theta$ is RTP and $r$ is the real rate of interest. This condition is a foundation of both static and dynamic economic studies. The mechanisms of both $\theta$ and $r$ are equally important. Particularly, RTP is an essential element in expectations of economic activities because RTP is the discount factor for future utility. In addition, RTP has been regarded as changeable even over short periods (e.g., Uzawa, 1968; Epstein and Hynes, 1983; Lucas and Stokey, 1984; Parkin, 1988; Obstfeld, 1990; Becker and Mulligan, 1997). Furthermore, households behave based on the expected RTP of the representative household (RTP RH) (Harashima, 2014, 2016). That is, changes in RTP and the expected RTP RH can be an important source of economic fluctuations.

### 2.2 The model

The model in this paper is based on the models in Harashima (2004, 2009, 2013a) and assumes non-cooperative, identical, and infinitely long living households, and that the number of households is sufficiently large. Each of the households equally maximizes the expected utility

$$
E_{0} \int_{0}^{\infty} \exp (-\theta t) u\left(c_{t}\right) d t
$$

subject to

$$
\frac{d k_{t}}{d t}=f\left(A, k_{t}\right)-c_{t}
$$

where $y_{t}, c_{t}$, and $k_{t}$ are production, consumption, and capital per capita in period $t$, respectively; $A$ is technology and constant; $u$ is the utility function; $y_{t}=f\left(A, k_{t}\right)$ is the production function; and $E_{0}$ is the expectations operator conditioned on the agents' period 0 information set. $y_{t}, c_{t}$, and $k_{t}$ are monotonically continuous and differentiable in $t$, and $u$ and $f$ are monotonically continuous functions of $c_{t}$ and $k_{t}$, respectively. All households initially have an identical amount of financial assets equal to $k_{t}$, and all households gain the identical amount of income $y_{t}=f\left(A, k_{t}\right)$ in each period. It is assumed that $\frac{d u\left(c_{t}\right)}{d c_{t}}>0$ and $\frac{d^{2} u\left(c_{t}\right)}{d c_{t}^{2}}<0$; thus, households are risk averse. In addition, $\frac{\partial f\left(A, k_{t}\right)}{\partial k_{t}}>0$ and $\frac{\partial^{2} f\left(k_{t}\right)}{\partial k_{t}^{2}}<0$. Both technology (A) and labor supply are assumed to be constant; that is, there is no technological progress or population increase. It is also assumed that there is no depreciation of capital.

### 2.3 A Nash equilibrium of a Pareto inefficient path

The effects of an upward shift in RTP are shown in Figure 1. Suppose first that the economy is at steady state before the shock. After the upward RTP shock, the vertical line $\frac{d c_{t}}{d t}=0$ moves to the left (from the solid vertical line to the dashed vertical line in Figure 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point $Z$. After the jump, consumption proceeds on the Pareto efficient saddle path (the posterior saddle path) from point $Z$ to the lower steady state after the shock (the posterior steady state). As a result, negative economic growth rates continue for a long period, but unemployment rates will not increase and resources will not be destroyed or left idle. Note that an increase in household consumption means consuming the part capital indicated by the gap between the posterior saddle path (the thin dashed curve) and production (the bold solid curve) for each $k_{t}$, which initially is the gap between point $Z$ and $W .{ }^{1}$

Figure 1: An upward RTP shock. All terms are defined in the text.
$c_{t}$


[^1]However, this discontinuous jump to $Z$ will be uncomfortable for risk-averse households that wish to smooth consumption. Households may instead chose a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line), although this shortcut is not Pareto efficient. The mechanism for why they are very unlikely to jump consumption is explained in Harashima (2004, 2009, 2013a) and also in the Appendix. Because households are risk averse and want to smooth consumption, and are also intrinsically non-cooperative, they behave strategically in game theoretic situations. Because of these features, when households strategically consider whether or not the jump is better for them (i.e., they are in a game theoretic situation), they will generally conclude that they obtain a higher expected utility if they do not jump. Hence, households will not actually choose this path and instead will choose a different transition path to the steady state (e.g., the bold dashed curve). Because this transition path is not on the posterior saddle path, it is not Pareto efficient (I call this transition path a "Nash equilibrium of a Pareto inefficient path" or more simply a "Pareto inefficient transition path"). Therefore, the excess resources indicated by the gap between the posterior saddle path (the thin dashed curve) and the Pareto inefficient transition path (the bold dashed curve) for each $k_{t}$ (initially, the gap between points $Z$ and $X$ ) will be destroyed or left idle. Unemployment rates will increase sharply and stay high for a long period.

## 3 SHOULD THE GOVERNMENT FISCALLY INTERVENE?

### 3.1 The government's options

### 3.1.1 The three options

When households choose a Nash equilibrium of a Pareto inefficient path, the government basically has three options: (1) do not intervene, (2) increase government consumption, and (3) cut taxes.

If Option (1) is chosen, the gap between the posterior saddle path and the Pareto inefficient transition path (initially the gap between points $Z$ and $W$ ) is not filled by any demand. Therefore, unemployment rates increase sharply and huge amounts of resources are destroyed or left idle. High unemployment rates and destruction of resources will continue until the economy reaches the posterior steady state.

If Option (2) is chosen, government consumption is increased to fill the demand gap between the posterior saddle path and the Pareto inefficient transition path, where government consumption is indicated on a per capita basis similar to the other variables. Suppose for simplicity that government consumption is zero before the shock. With increases in government consumption, the path of the sum of government and household consumption (hereafter "combined consumption") can be equal to the posterior saddle path.

Conceptually, government consumption is the collective consumption of households through government expenditures, for example, spending on various kinds of administrative services that households receive. Therefore, increases in government consumption can be substituted for decreases in household consumption. Nevertheless, government consumption will not directly generate utility in households. In this sense, increases in government consumption may be interpreted as forced increases in household consumption. Even if households do not want these increases in government consumption, however, the increases will work to increase aggregate demand. Option (2) therefore indicates a measure to compulsorily fill the gap between aggregate demand and supply, even against households' will, when the economy proceeds on a Pareto inefficient transition path. Notice that the excess resources cannot be used for investments because the economy would otherwise deviate from a path to the steady state.

If Option (3) is chosen, households' disposable incomes will increase, but if the Ricardian equivalence holds, they will still proceed on a Pareto inefficient transition path. Because household consumption does not change, high unemployment rates and destruction of a huge amount of resources continue as in Option (1). Because there is a huge amount of excess capital, no additional investment will be made. Nevertheless, if the Ricardian equivalence does not hold, tax cuts may increase household consumption at least temporarily. Therefore, the validity of Option (3) depends on the validity of the Ricardian equivalence. If households are sufficiently rational, the Ricardian equivalence will basically hold at least in the long run. Therefore, even if tax cuts are effective, they will be effective only in the short run, and these short run effects will be reversed because the Ricardian equivalence will hold in the long run.

### 3.1.2 Financing

In Option (3), tax cuts are financed by borrowing from households. In Option (2), an increase in the government consumption is financed by borrowing from or tax increases on households. Nevertheless, financing by borrowing will be preferred in Option (2) because the Ricardian equivalence may not necessarily hold in the short run. If the Ricardian equivalence does not hold, increases in taxes may increase unemployment rates and thereby the main aim of Option (2) cannot be fully achieved. Therefore, it is highly likely that an increase in government consumption will be financed by government borrowing, and therefore borrowing is assumed in this paper. However, financing by borrowing requires tax increases in the future to pay off the debt with interest. Options (2) and (3) assume that necessary future tax increases are fully implemented by the government.

In addition, it is assumed that a government borrows money only from its own people, that is, not from foreigners because foreign borrowing means that foreigners also intervene in addition to the government, and such intervention is beyond the scope of this paper.

### 3.2 Comparison among options

(1) Economic growth rate

Because production and consumption at the posterior steady state are lower than those at the prior steady state, the rate of economic growth is equally negative during the transition in the three options except for a subordinate option of Option (2), in which, as will be shown in Section 4, it is zero. Nevertheless, there actually still will be steady technological progress (remember that no technological progress is assumed in the model), and thereby the actual rates of growth will not necessarily be negative or zero and may even be low but positive.
(2) Household utility

Households choose a Nash equilibrium of a Pareto inefficient path equally in the three options. Therefore, the utilities of households are basically same in the three options.

## (3) Unemployment

In Options (1) and (3), unemployment rates will rise sharply and stay high for a long period. In contrast, in Option (2), high unemployment rates can be avoided because the gap of demand is filled by increases in government consumption and thereby no resources are destroyed or left idle.
(4) Government debt

In Option (1), government debt does not increase because the government does not borrow additional money, but in Options (2) and (3), government debt will increase because of continuous financing by borrowing. However, if taxes are raised properly to pay off the debt in the future, government debt will stabilize in some future period.

### 3.3 Government debt

### 3.3.1 Is the government debt sustainable?

The usual arguments on sustainable government debts (e.g., Hamilton and Flavin, 1986; Bohn, 1995) are not applicable to the government debts in Options (2) and (3) because households proceed on an "unusual" Pareto inefficient transition path, so an alternative approach is necessary. Let $d_{t}$ be per capita "extra" government debts in period $t$ that are accumulated in Option (2) or (3). Because all $d_{t}$ are owned by households as assumed above, $d_{t}$ also indicates the financial assets of households, and the other household assets (other than $d_{t}$ ) are ignored for simplicity. In the future, $d_{t}$ is redeemed with interest, but the redemption takes a long time. Because the Ricardian equivalence will hold in the long run, it is assumed that household consumption is not influenced by $d_{t}$. Let $z_{t}$ be per capita taxes to redeem a part of $d_{t}$ in period $t$ and also let $g_{t}$ be additional government borrowing in Option (2) or (3) in period $t$. In Option (2),

$$
\begin{equation*}
y_{t}=c_{t}+g_{t} \tag{1}
\end{equation*}
$$

and in Option (3),

$$
\begin{equation*}
y_{t} \geq c_{t}+g_{t} \tag{2}
\end{equation*}
$$

for any $t$ because no new investment is made in Options (2) and (3) and the household assets other than the government bonds are ignored; $y_{t}$ and $c_{t}$ are per capita income and consumption of households in period $t$. If the condition

$$
\begin{equation*}
r_{t} d_{t}+g_{t} \leq z_{t} \tag{3}
\end{equation*}
$$

is satisfied indefinitely in a certain future period, government debt never explodes; that is, it is sustainable where $r_{t}\left(0 \leq r_{t}<1\right)$ is the real interest rate. By equality (1) and inequality (3), the condition for sustainability in Option (2) is

$$
\begin{equation*}
y_{t}-c_{t}+r_{t} d_{t t} \leq z_{t} \tag{4}
\end{equation*}
$$

By inequalities (2) and (3), if inequality (4) is satisfied indefinitely in a certain future period, government debt is also sustainable in Option (3).

Because the household assets other than $d_{t}$ are ignored, the sum of a household's income and assets is

$$
d_{t}+y_{t}-c_{t}
$$

If the sum of a household's income and assets exceeds $z_{t}$, that is, if

$$
\begin{equation*}
z_{t}<d_{t}+y_{t}-c_{t} \tag{5}
\end{equation*}
$$

then $z_{t}$ can be imposed in the sense that households have enough resources to fully pay taxes. Hence, by inequalities (4) and (5), if

$$
\begin{equation*}
r_{t} d_{t}<d_{t} \tag{6}
\end{equation*}
$$

is satisfied, taxes that satisfy the condition for sustainable debts can be imposed. Here, because $0 \leq r_{t}<1$, then inequality (6) always holds. Therefore, for any $d_{t}$, there always exists $z_{t}$ that satisfies inequality (3) indefinitely in a certain future period. That is, the government debt can be sustainable for any $d_{t}$, and even if $d_{t}$ becomes extremely large, the debt can be sustainable. Consider an extreme example. If a government collects taxes that are equivalent to $d_{t}$ from a household's financial assets in a period, the government's debts are eliminated completely all at once. That is, any $d_{t}$ can be sustainable.

Such an extreme tax will not actually be imposed, but if $d_{t}$ exceeds a certain amount such that

$$
y_{t}<r d_{t} \leq z_{t},
$$

(i.e., if taxes exceed income), then they need to be collected from a part of a household's holdings of $d_{t}$. If households well know the possibility of a tax on $d_{t}$ in the future, they will not regard their accumulated financial assets corresponding to $d_{t}$ as their "real" assets in the sense they can be freely used for consumption even though $d_{t}$ may be extremely large. In addition, because any $d_{t}$ can be sustainable, the tax increase can be started even after all the excess capital is eliminated. Hence, a huge amount of government debt can remain even if there is no excess capital.

Finally, it is important to note that the increased tax revenues should not be used to finance increases in government consumption for purposes other than dealing with the excess capital. The increased taxes should be used only to pay down $d_{t}$ (with interest) because the economy otherwise deviates from the steady state.

### 3.3.2 How large can government debt be?

Any $d_{t}$ can be sustainable but only if a government properly raises taxes and $r_{t} d_{t} \leq z_{t}$ is satisfied indefinitely in a certain future period. The question arises, however, when is "a certain future period"? The time at which taxes are raised is indeterminate in the discussion in the previous section. The tax increase can be postponed almost indefinitely if taxes will certainly be raised eventually. This indeterminacy may generate a political struggle because people intrinsically dislike tax increases, and opposition parties will utilize people's anti-tax sentiment as ammunition to attack the government. Opposition parties will appeal to people that a tax increase is not necessary at present and that it will only generate a recession because the Ricardian equivalence will not hold in the short run. The government may not sufficiently refute this argument and persuade people that the current level of government debt is unsustainable, because any $d_{t}$ can be sustainable. The incentive for the government to raise taxes to reduce $d_{t}$ will therefore be weak.

Is there a problem, however, if $d_{t}$ becomes extremely large? As shown in Section 3.2.1, other things being equal, any $d_{t}$ can be sustainable, but if something changes and affects the sustainability as $d_{t}$ becomes larger, a large $d_{t}$ will not actually be sustainable. One possible factor that may change as $d_{t}$ becomes larger is uncertainty. If the tax increase has been postponed for a long period, questions about the ability of the government to govern the nation and run the economy will arise. Faced with an extremely large $d_{t}$, people may begin to suspect that their government cannot do what it should do. Hence, uncertainty about the ability of the government will increase, and increased uncertainty about the government's ability means that the government's performance in the future is no longer a certainty.

It has been argued that good institutions, including governments, enhance economic growth (e.g., Knack and Keefer, 1995; Mauro, 1995; Hall and Jones, 1999; Acemoglu et al., 2001, 2002; Easterly and Levine, 2003; Dollar and Kraay, 2003; Rodrik et al., 2004). Acemoglu et al. (2005) conclude that differences in economic institutions are empirically and theoretically
the fundamental cause of differences in economic development. ${ }^{2}$ It is therefore highly likely that a government's ability is an important determinant of total factor productivity, that is, levels of production and consumption. Therefore, if uncertainty about the ability of a government increases, household's expected variances of production and consumption will also increase. Larger variances of production and consumption mean more uncertainty about the entire future economy. That is, as $d_{t}$ increases, household uncertainty about the entire future economy increases.

An important consequence of increases in uncertainty about the entire future economy is an increase in household RTP. The concept of a temporally varying RTP has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930; Uzawa, 1968; Lawrance, 1991; Becker and Mulligan, 1997). In addition, uncertainty has been regarded as a key factor that changes RTP. Fisher (1930) argued that uncertainty, or risk, must naturally influence RTP, and higher uncertainty tends to raise RTP. Harashima (2004, 2009) showed a mechanism of how an increase in uncertainty leads to an increase in RTP by constructing an endogenous RTP model where uncertainty is defined by the stochastic dominance of the distribution of steady-state consumption. Increases in uncertainty will increase RTP RH. An increase in RTP RH indicates an increase in the real interest rate at steady state and consequently a decrease in production and consumption at the steady state because RTP RH is equal to the real interest rate at steady state in Ramsey-type growth models. That is, it is likely that as $d_{t}$ increases, long-run production and consumption will decrease.

Considering the effect of $d_{t}$ on RTP RH and on long run production and consumption, therefore, a government will not have to postpone the a tax increase for a long period and to accumulate an extremely large $d_{t}$. Nevertheless, the scale of the effect of $d_{t}$ on RTP RH is unclear. It may be small and take a long period before households clearly recognize the negative effect of a large $d_{t}$ on RTP RH. Hence, the exact upper limit of $d_{t}$ is unclear, so there will still be much room for a government with regard to the timing and scale of tax increases.

When the long run negative effect of a huge $d_{t}$ on the expected household utility becomes larger than the short run effect of deviation from the Ricardian equivalence on the expected household utility, taxes should be raised. However, it may be difficult to judge which is currently larger. On the other hand, if the negative effect of the short run deviation from the Ricardian equivalence can be controlled such that it remains very small, it will be better to raise taxes even for small $d_{t}$. In this sense, it may be a good idea to raise the tax rate by a very small percentage point amount in every period, for example, by $0.5 \%$ per year. Because this tax increase is very small in each period, the negative effect of any short run deviation from the Ricardian equivalence can be controlled such that it is also very small in each period.

There is another relatively minor problem associated with extremely large $d_{t}$. As $d_{t}$ increases, the amount of necessary future tax increases (as shown in Section 3.3.1) will eventually exceed income $\left(y_{t}\right)$. Therefore, taxes need to be imposed not only on income but also on household's financial assets corresponding to $d_{t}$. However, large taxes on financial assets may be less easy to implement than other types of taxes both practically and politically. Nevertheless, an inheritance tax may be relatively easy to implement, and therefore it will be important as taxes on household's financial assets.

### 3.3.3 Price stability

It has been argued that a large amount of government debt will result in high inflation (Sargent and Wallace, 1981). Fiscal theory of price level particularly emphasizes this mechanism (Leeper, 1991; Sims, 1994, 1998; Cochrane, 2005; Woodford, 2001). However, Harashima (2006) showed that the relation between the government debts and inflation is not simple and presented a model that explains the law of motion for inflation considering government debt. The model in Harashima (2006) indicates that a large amount of government debts does not result in high

[^2]inflation as long as the central bank is sufficiently independent. Inflation will not be affected by temporary increases in government expenditure and consequent future taxes. As a result, if the central bank is sufficiently independent, the government can implement Option (2) without worrying about an outbreak of high inflation.

### 3.4 Evaluation

As shown in Section 3.2, the rate of economic growth in the three options is equally negative until arriving at the steady state, and household utilities are basically same in the three options. On the other hand, unemployment rates will rise sharply and stay high for a long period in Options (1) and (3), but not in Option (2). As argued in Section 3.3, the extra government debts are sustainable if the government properly increases taxes in the future. If the future tax increase is properly implemented, therefore, Option (2) is favorable to Options (1) and (3) because unemployment rates do not rise.

## 4 HOW SHOULD THE GOVERNMENT INCREASE ITS CONSUMPTION?

### 4.1 Subordinate options in Option (2)

Option (2) is the best choice, but how should the government increase its consumption? There are two basic subordinate options in Option (2).

Option (2-1): Increase government consumption in order for the combined consumption to jump up to point $Z$ and then proceed on the posterior saddle path to the posterior steady state.
Option (2-2): Increase government consumption for the combined consumption to jump up to point $W$, and then stay at point $W$.

Remember that combined consumption indicates the sum of government and household consumptions. Option (2-1) indicates that the government intervenes so as to make the combined consumption proceed on the posterior saddle path and eventually reach the posterior steady state, and Option (2-2) indicates that it intervenes so as to make the production and combined consumption stay at the prior steady state (i.e., at point $W$ ) forever. Note that, as noted in Section 3.1.1, excess resources cannot be used for investments because the economy would otherwise deviate from the posterior saddle path in Option (2-1) and from point $W$ in Option (2-2).

### 4.2 Option (2-1)

### 4.2.1 Basic features

When a government chooses Option (2-1), each household may change its consumption path in response to the government's action, but it is highly likely that households will still proceed on a Pareto inefficient transition path because the households' expected utilities are not affected by the increase in government consumption. Here, a gap between the posterior saddle path (the thin dashed curve in Figure 1) and production (the bold solid curve) for each $k_{t}$ indicates excess capital. Excess capital needs to be "consumed" for the economy to be on the posterior saddle path. ${ }^{3}$ Option (2-1) means that excess capital is consumed by the government. In addition, to be on the posterior saddle path, government consumption needs to be increased not only to consume excess capital but also to substitute for a reduction in household consumption that is the source of the excess capital. That is, the government needs to consume not only the gap between the posterior saddle path and production (i.e., excess capital), but also the gap between

[^3]production and the Pareto inefficient transition path while the economy proceeds from the prior steady state to the posterior steady state. Because of the increase in government consumption, the economy proceeds on the posterior saddle path and thereby high and persistent unemployment rates are avoided.

### 4.2.2 Subordinate options

However, how does a government "consume" such a large quantity of excess resources, most of which were originally produced as capital? There are three basic subordinate options: Options (2-1-a), (2-1-b), and (2-1-c).

The easiest way for a government to consume the excess resources is simply to buy them from firms and dispose of them (Option (2-1-a)). "Dispose of" in this case includes not only eliminating them but also leaving them unused forever or constructing useless infrastructure. It will also mean giving laborers busy work, including the classic example of "having workers dig holes and then fill them back up." These activities do not generate any utility for households, but they can be interpreted as a kind of "consumption" in the broad sense that the products purchased are intentionally made unusable. High unemployment rates can be avoided, but huge amounts of resources are systematically and continuously disposed of and negative growth rates continue for a long period.

Disposing of the excess resources in Option (2-2-a) is different from destroying them in Option (1) because the owners of the excess resources lose them without compensation in Option (1), but sell them to the government in Option (2-1-a). The excess resources are equally eliminated in both options, but nothing remains in the hands of the former owners or the government in Option (1), whereas financial assets and debts remain in the hands of the former owners and government, respectively, in Option (2-1-a).

Another way to consume the excess resources is to export them to other countries at lower prices than the prevailing international prices (Option (2-1-b). This is not "consumption" in the literal sense, but it can be interpreted as a sort of consumption in that exports are an element of demand. The government does not necessarily need to directly export the excess resources. Instead, it can indirectly support exports by directly subsidizing firms or through various kinds of regulations. An important problem with this option is that other countries may not accept the excessive exports. This option clearly means setting prices that are far lower than the costs of production (i.e., dumping) on a large scale. Other countries would not be likely to stay silent on this issue and would likely take countermeasures, for example, by imposing high anti-dumping customs. Therefore, Option (2-1-b) will generally not be adopted in a democratic country.

There is one more important subordinate option. With minor modifications, capital inputs can be used to produce arms and munitions. Hence, the necessary increase in government consumption can easily be achieved by a large military buildup (Option (2-1-c)). An important problem with this option is that a unilateral excessive military buildup will greatly worsen international relations and increase political and military tensions among countries. Therefore, in a democratic country, Option (2-1-c) will generally not be adopted.

### 4.3 Option (2-2)

### 4.3.1 Basic features

For the same reason as given for Option (2-1), it is highly likely that households also proceed on a Pareto inefficient transition path in Option (2-2). When households proceed on this path, if the government does nothing, a part of the capital that is used to produce products corresponding to households' reduction in consumption becomes excess capital and will be destroyed, but if the government purchases and consumes these unconsumed products, the capital need not be destroyed and the level of capital will remain the same in the next period. If the government purchases and consumes the unconsumed products in every period, capital will continue to stay at the same level indicated by point $W$. The phenomenon where capital is prevented from being
reduced by government intervention may be interpreted as keeping so-called "zombie" firms alive. As in Option (2-1), high unemployment rates can be avoided, but unlike in Option (2-1), the growth rate is not negative. Rather, it is zero because the economy stays at point $W$ forever.

An important difference between Options (2-1) and (2-2) is that, unlike Option (2-1), capital is not consumed by the government in Option (2-2), but households' reduction in consumption is equally substituted by an increase in government consumption in both options. That is, in Option (2-2), the government consumes only the gap between production at point $W$ and the Pareto inefficient transition path (bold dashed curve) and does not consume the gap between the posterior saddle path (thin dashed curve) and production at point $W$ (i.e., capital). As a result, production and capital remain at point $W$ forever in Option (2-2).

### 4.3.2 Subordinate options

Option (2-2) also consists of three basic subordinate options depending on what path is chosen at point $W$ : Options (2-2-a), (2-2-b), and (2-2-c). As was the case with Option (2-1-a), the easiest way for a government to consume excess resources is simply to buy them from firms and dispose of them (Option (2-2-a)). As with Options (2-1-b) and (2-1-c), the necessary jump of the government consumption can be achieved by exporting the excess resources (Option (2-2-b)) or by a military buildup (Option (2-2-c)). However, for the same reasons as given for Options (2-1-b) and (2-1-c), Options (2-2-b) and (2-2-c) will generally not be adopted in a democratic country.

### 4.4 Comparison and evaluation

Section 4.3 indicates that the only feasible options are (2-1-a) and (2-2-a). On major issues, commonalities and differences between the two options are as follows.
(1) Period of government intervention

In Option (2-1-a), excess capital decreases gradually and eventually becomes zero when the economy arrives at the posterior steady state. ${ }^{4}$ Hence, the period of transition and government intervention is definite. In Option (2-2-a), however, the economy never approaches the posterior steady state. Hence, the government intervention never ends.
(2) Scale of government intervention

Because government consumption needs to be initially increased to point $Z$ in Option (2-1-a), the scale of intervention is initially much larger in Option (2-1-a) than in Option (2-2-a). However, in Option (2-1-a), excess capital gradually decreases and eventually reaches the level of the posterior steady state, and thereby the necessary increase in government consumption decreases to zero as the economy approaches the posterior steady state. On the other hand, in Option (2-2-a), the necessary increase in government consumption increases as household consumption gradually decreases to the level at the posterior steady state. In sum, the scale of intervention is initially larger in Option (2-1-a) than it is Option (2-2-a), but this relation will be reversed in some future period.
(3) Growth rates during the transition

In Option (2-1-a), the growth rates are negative, whereas in Option (2-2-a), they are zero.
(4) Household utility

In both options, household consumption proceeds on the same Pareto inefficient transition path. In addition, the Ricardian equivalence holds in the long run. Therefore, the utilities that

[^4]households will obtain from the stream of consumption after the shock are almost the same in both cases.
(5) Unemployment

In both options, unemployment rates do not increase.
(6) Government debt

In both options, a large amount of government debt accumulates. However, if the government properly increases taxes in the future, the debt will stabilize at some level in both options.

Although the period and scale of government interventions differ between the two options, these differences basically do not matter to household optimality. Therefore, because the only difference in the evaluated criteria is that growth rates are higher in Option (2-2-a), Option (2-2-a) is considered to be more favorable than Option (2-1-a).

### 4.5 Technological progress

Although Option (2-2-a) is the best, it has its drawbacks. Huge amounts of resources need to be disposed of in the name of the government consumption forever. Although this is rational from an economic point of view, it may not be environmentally or ethically reasonable. If there is a way to reduce the amount of discarded resources, that is, reduce excess capital, Option (2-2-a) could be much better. It is impossible to find that way within the framework discussed in the previous sections, but if the assumption on technological progress is loosened, it may be possible.

Thus far, I have assumed no technological progress, but in reality, technologies steadily progress. In addition, technological progress basically requires additional increases in capital. Instead of adding capital, however, the new capital that is embedded in new technologies can be introduced by using part of the excess capital. As a result, the amount of excess capital is gradually reduced as part of the process of technological progress. Of course, not all of the excess capital can be easily replaced in each period, but most of it should be able to be replaced in the long run.

With the gradual replacement of the excess capital through technological progress, the excess capital will eventually be fully eliminated and the government intervention will end. Note nevertheless that this elimination process will take a long time. In addition, the economic growth caused by technological progress will be slower because part of the increase in capital required by technological progress is being replaced with a reduction in excess capital. The economy will therefore grow more slowly because of the relatively slower growth of capital

## 5 DISCUSSION

### 5.1 Japan since the 1990s

Japan has experienced low, occasionally negative, growth rates since the 1990s, even though the Japanese government has spent huge amounts of money to stabilize its economy by issuing similarly huge amounts of government bonds. At the same time, the debts of the Japanese government have greatly increased. Japan's experience seems to be very similar to the consequences predicted when Option (2-2-a) is chosen. This similarity implies that the stagnation of the Japanese economy since the 1990s was caused by an upward RTP shock, and the Japanese government chose Option (2-2-a) as the countermeasure to the shock. Harashima (2016) examines this possibility theoretically and empirically and concludes that RTP RH of Japan rose $2-3$ percentage points in the early 1990s, and this upward shift of RTP RH was the cause of the stagnation of Japanese economy since the 1990s.

If the Japanese government had not chosen Option (2-2-a) and had instead chosen Option (1), Japan would have experienced a significantly more severe recession, possibly similar to the Great Depression of the 1930s. Production would have decreased and unemployment rates would have increased far more than they did actually. Therefore, the Japanese government may be praised for choosing the best option when facing a large upward shift of RTP RH. However, the Japanese government should keep in mind that Option (2-2-a) is only the best option if the government properly increases taxes to redeem the debts at some point in the future.

### 5.2 The Great Depression and World War II

Many hypotheses on the causes of the Great Depression in the 1930s have been presented, but no consensus has been reached. The phenomena observed during the Great Depression are very similar to those predicted when Option (1) is chosen; that is, the growth rates were negative and unemployment rates rose sharply. In addition, this agonizing situation was prolonged. Here, I have indicated that the best option to tackle such a situation is to adopt Option (2-2-a), but large discretionary fiscal interventions by governments were generally seen as taboo in that period. Government expenditures were increased only to a limited extent in the United States with the introduction of the New Deal, and the Great Depression persisted.

However, the U.S. economy recovered in 1940s after government consumption was greatly increased to build up the military in the face of the outbreak of World War II. It is likely that the U.S. government unintentionally or compulsorily chose Option (2-1-c) or (2-2-c). Unemployment rates declined and destroying or disposing of resources stopped as predicted by both options. In this case, it appears that the taboo against discretionary fiscal intervention was broken because of the threat and outbreak of a large-scale war.

Similar phenomena were observed in Germany. Germany was one of the hardest-hit economies by the Great Depression, but after the Nazis took power in 1933, the German economy recovered quickly and sharply. The government of Nazi Germany significantly intervened in various aspects of the German economy. This intervention eliminated the large-scale Pareto inefficiency that was generated by the Great Depression. In particular, the German government greatly built up its military so it is likely that Option (2-1-c) or (2-2-c) was adopted to restore the German economy.

## 6 CONCLUDING REMARKS

If the steady state is shifted downwards by a fundamental shock, each household must change its consumption path to one that diminishes gradually to the posterior steady state. Because consumption decreases, a recession begins. In this case, if households increase their consumption discontinuously to a point on the posterior saddle path and then follow that to the posterior steady state, Pareto efficiency is held and unemployment rates do not rise. However, households will not behave like this because it does not give them the highest expected utility. Households are risk averse and dislike unsmooth and discontinuous consumption. Instead, households will choose a Nash equilibrium of a Pareto inefficient path as the optimal consumption path. Because of its Pareto inefficiency, the unemployment rate will increase sharply and stay high for a long period.

In this paper, I examined whether discretionary fiscal policy is necessary if this type of recession occurs, and if it is necessary, how it should be implemented. Particularly, the fiscal policy for a Nash equilibrium of a Pareto inefficient path caused by an upward shock on RTP was examined. In this case, a government has three options: (1) do not intervene, (2) increase government consumption, and (3) cut taxes. Option (2) has several subordinate options. I compared and evaluated these options and concluded that increasing government consumption until the demand meets the present level of production and maintaining this fiscal policy is the best option. The accompanying huge government debts can be sustainable even though they are
extremely large if the government properly increases taxes in the future. In this option, large government debts play an essential role as a buffer against the negative effects of the shock.

## APPENDIX

## A Nash equilibrium of a Pareto inefficient path

## A1 Model with non-cooperative households ${ }^{5}$

## A1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path. Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower levels of production and consumption than before the shock, which is consistent with the phenomena actually observed in a recession.

Although the rate of time preference (RTP) is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because RTP is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant RTP exhibit excellent tractability (see Samuelson, 1937). However, RTP has been naturally assumed and actually observed to be time-variable. The concept of a time-varying RTP has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant RTPs by nature and that economic and social factors affect the formation of RTPs. Their arguments indicate that many incidents can affect and change RTP throughout a person's life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of RTP, and showed that RTP was as volatile as technology and leisure preference.

## A1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical, and infinitely long living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$
E_{0} \int_{0}^{\infty} \exp (-\theta t) u\left(c_{t}\right) d t,
$$

subject to

$$
\frac{d k_{t}}{d t}=f\left(A, k_{t}\right)-\delta k_{t}-c_{t}
$$

where $y_{t}, c_{t}$, and $k_{t}$ are production, consumption, and capital per capita in period $t$, respectively; $A$ is technology and constant; $u$ is the utility function; $y_{t}=f\left(A, k_{t}\right)$ is the production function; $\theta(>0)$ is RTP; $\delta$ is the rate of depreciation; and $E_{0}$ is the expectations operator conditioned on the agents' period 0 information set. $y_{t}, c_{t}$, and $k_{t}$ are monotonically continuous and differentiable in $t$, and $u$ and $f$ are monotonically continuous functions of $c_{t}$ and $k_{t}$, respectively. All households initially have an identical amount of financial assets equal to $k_{t}$, and all households gain the identical amount of income $y_{t}=f\left(A, k_{t}\right)$ in each period. It is assumed

[^5]that $\frac{d u\left(c_{t}\right)}{d c_{t}}>0$ and $\frac{d^{2} u\left(c_{t}\right)}{d c_{t}^{2}}<0$; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion utility function
\[

$$
\begin{array}{ll}
u\left(c_{t}\right)=\frac{c_{t}^{1-\gamma}}{1-\gamma} & \text { if } \gamma \neq 1 \\
u\left(c_{t}\right)=\ln \left(c_{t}\right) & \text { if } \gamma=1
\end{array}
$$
\]

where $\gamma$ is a constant and $0<\gamma<\infty$. In addition, $\frac{\partial f\left(A, k_{t}\right)}{\partial k_{t}}>0$ and $\frac{\partial^{2} f\left(k_{t}\right)}{\partial k_{t}^{2}}<0$. Both technology $(A)$ and labor supply are assumed to be constant.

The effects of an upward shift in RTP are shown in Figure A1. Suppose first that the economy is at steady state before the shock. After the upward RTP shock, the vertical line $\frac{d c_{t}}{d t}=0$ moves to the left (from the solid vertical line to the dashed vertical line in Fig. 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point $Z$. After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point $Z$ to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to $Z$ may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household's maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward shift in RTP. The first is a jump option, $J$, in which a household's consumption jumps to $Z$ and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option, $N J$, in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure A1. The household that chooses the $N J$ option reaches the posterior steady state in period $s(\geq 0)$. The difference in consumption between the two options in each period $t$ is $b_{t}(\geq 0)$. Thus, $b_{0}$ indicates the difference between $Z$ and the prior steady state. $b_{t}$ diminishes continuously and becomes zero in period $s$. The $N J$ path of consumption $\left(c_{t}\right)$ after the shock is monotonically continuous and differentiable in $t$ and $\frac{d c_{t}}{d t}<0$ if $0 \leq t<s$. In addition,

$$
\begin{array}{ll}
\bar{c}<c_{t}<\hat{c}_{t} & \text { if } 0 \leq t<s \\
c_{t}=\bar{c} & \text { if } 0 \leq s \leq t,
\end{array}
$$

where $\hat{c}_{t}$ is consumption when proceeding on the posterior Pareto efficient saddle path and $\bar{c}$ is consumption in the posterior steady state. Therefore,

$$
\begin{array}{ll}
b_{t}=\hat{c}_{t}-c_{t}>0 & \text { if } 0 \leq t<s \\
b_{t}=0 & \text { if } 0 \leq s \leq t .
\end{array}
$$

It is also assumed that, when a household chooses a different option from the one the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption $\left(b_{t}\right)$ before period $s$ between that household and the other households is reflected in consumption after period $s$. That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption in each period after period $s$. The exact functional form of the addition (or subtraction) is shown in Section A1.4.

## A1.3 Firms

Unutilized products because of $b_{t}$ are eliminated quickly in each period by firms because holding them for a long period is a cost to firms. Elimination of unutilized products is accomplished by discarding the goods or preemptively suspending production, thereby leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital used to generate the unutilized products. Disposing of the excess capital is rational for firms because the excess capital is an unnecessary cost, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the $N J$ path (that is, if all households choose the $N J$ option), firms dispose of all of the remaining excess capital that generates $b_{t}$ and adjust their capital to the posterior steady-state level in period $s$, which also corresponds to households reaching the posterior steady state. Thus, if the economy proceeds on the $N J$ path, capital $k_{t}$ is

$$
\begin{array}{ll}
\bar{k}<k_{t} \leq \hat{k}_{t} & \text { if } 0 \leq t<s \\
k_{t}=\bar{k} & \text { if } 0 \leq s \leq t
\end{array}
$$

where $\hat{k}_{t}$ is capital per capita when proceeding on the posterior Pareto efficient saddle path and $\bar{k}$ is capital per capita in the posterior steady state.

The real interest rate $i_{t}$ is

$$
i_{t}=\frac{\partial f\left(A, k_{t}\right)}{\partial k_{t}}
$$

Because the real interest rate equals RTP at steady state, if the economy proceeds on the $N J$ path,

$$
\begin{array}{ll}
\tilde{\theta} \leq i_{t}<\theta & \text { if } 0 \leq t<s \\
i_{t}=\theta & \text { if } 0 \leq s \leq t
\end{array}
$$

where $\tilde{\theta}$ is RTP before the shock and $\theta$ is RTP after the shock. $i_{t}$ is monotonically continuous and differentiable in $t$ if $0 \leq t<s$.

## A1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of the $J$ or $N J$ path. Let Jalone indicate that the household chooses option $J$, but the other households choose option $N J$; $N J a l o n e ~ i n d i c a t e ~ t h a t ~ t h e ~ h o u s e h o l d ~ c h o o s e s ~ o p t i o n ~ N J, ~ b u t ~ t h e ~ o t h e r ~ h o u s e h o l d s ~ c h o o s e ~ o p t i o n ~$ $J$; Jtogether indicate that all households choose option $J$; and NJtogether indicate that all
households choose option $N J$. Let $p(0 \leq p \leq 1)$ be the subjective probability of a household that the other households choose the $J$ option (e.g., $p=0$ indicates that all the other households choose option $N J$ ). With $p$, the expected utility of a household when it chooses option $J$ is

$$
\begin{equation*}
E_{0}(J)=p E_{0}(\text { Jtogether })+(1-p) E_{0}(\text { Jalone }) \tag{A1}
\end{equation*}
$$

and when it chooses option $N J$ is

$$
\begin{equation*}
E_{0}(N J)=p E_{0}(N J a l o n e)+(1-p) E_{0}(\text { NJtogether }) \tag{A2}
\end{equation*}
$$

where $E_{0}($ Jalone $), E_{0}($ NJalone $), E_{0}$ (Jtogether), and $E_{0}(N J t o g e t h e r)$ are the expected utilities of the household when choosing Jalone, NJalone, Jtogether, and NJtogether, respectively. Given the properties of $J$ and $N J$ shown in Sections A1.2 and A1.3,

$$
\begin{align*}
E_{0}(J) & =p E_{0}\left[\int_{0}^{s} \exp (-\theta t) u\left(c_{t}+b_{t}\right) d t+\int_{s}^{\infty} \exp (-\theta t) u\left(\hat{c}_{t}\right) d t\right] \\
& +(1-p) E_{0}\left[\int_{0}^{s} \exp (-\theta t) u\left(c_{t}+b_{t}\right) d t+\int_{s}^{\infty} \exp (-\theta t) u(\bar{c}-\bar{a}) d t\right], \tag{A3}
\end{align*}
$$

and

$$
\begin{align*}
E_{0}(N J) & =p E_{0}\left[\int_{0}^{s} \exp (-\theta t) u\left(c_{t}\right) d t+\int_{s}^{\infty} \exp (-\theta t) u\left(\hat{c}_{t}+a_{t}\right) d t\right] \\
& +(1-p) E_{0}\left[\int_{0}^{s} \exp (-\theta t) u\left(c_{t}\right) d t+\int_{s}^{\infty} \exp (-\theta t) u(\bar{c}) d t\right] \tag{A4}
\end{align*}
$$

where

$$
\begin{equation*}
\bar{a}=\theta \int_{0}^{s} b_{r} \exp \int_{r}^{s} i_{q} d q d r \tag{A5}
\end{equation*}
$$

and

$$
\begin{equation*}
a_{t}=i_{t} \int_{0}^{s} b_{r} \exp \int_{r}^{s} i_{q} d q d r \tag{A6}
\end{equation*}
$$

and the shock occurred in period $t=0$. Figure A2 shows the paths of Jalone and NJalone. Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, in the case of Jalone, the economy almost proceeds on the $N J$ path. Similarly, in the case of NJalone, it almost proceeds on the $J$ path. If the other households choose the $N J$ option (Jalone or NJtogether), consumption after $s$ is constant as $\bar{c}$ and capital is adjusted to $\bar{k}$ by firms in period $s$. In addition, $a_{t}$ and $i_{t}$ are constant after $s$ such that $a_{t}$ equals $\bar{a}$ and $i_{s}$ equals $\theta$, because the economy is at the posterior steady state. Nevertheless, during the transition period before $s$, the value of $i_{t}$ changes from the value of the prior RTP to that of the posterior RTP. If the other households choose option $J$ (NJalone or Jtogether), however, consumption after $s$ is $\hat{c}_{t}$ and capital is not adjusted to $\bar{k}$ by firms in period $s$ and remains at $\hat{k}_{t}$.

As mentioned in Section A1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its
consumption in each period after period $s$. This is described by $a_{t}$ and $\bar{a}$ in equations (A3) and (A4), and equations (A5) and (A6) indicate that the accumulated difference in financial assets resulting from $b_{t}$ increases by compound interest between the period $r$ to $s$. That is, if the household takes the NJalone path, it accumulates more financial assets than each of the other $J$ households, and instead of immediately consuming these extra accumulated financial assets after period $s$, the household consumes the returns on them in every subsequent period. If the household takes the Jalone path, however, its consumption after $s$ is $\bar{c}-\bar{a}$, as shown in equation (A3). $\bar{a}$ is subtracted because the income of each household, $y_{t}=f\left(A, k_{t}\right)$, including the Jalone household, decreases equally by $b_{t}$. Each of the other $N J$ households decreases consumption by $b_{t}$ at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita; $k_{t}$ ) are kept equal to $\hat{k}_{t}$. The Jalone household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other $N J$ households, which results in the subtraction of $\bar{a}$ after period $s$.

## A2 Nash Equilibrium of Pareto Inefficiency Path ${ }^{6}$ <br> A2.1 Rational Pareto inefficient path

## A2.1.1 Rational choice of a Pareto inefficient path

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option $J$ is chosen as the path after the shock because it gives a higher expected utility than option NJ. Because there is no possibility of Jalone and NJalone if households are cooperative, then $E_{0}(J)=E_{0}($ Jtogether $)$ and $E_{0}(N J)=E_{0}($ NJtogether $)$. Therefore,

$$
\begin{aligned}
& E_{0}(J)-E_{0}(N J) \\
= & E_{0}\left[\int_{0}^{s} \exp (-\theta t) u\left(c_{t}+b_{t}\right) d t+\int_{s}^{\infty} \exp (-\theta t) u\left(\hat{c}_{t}\right) d t\right]-E_{0}\left[\int_{0}^{s} \exp (-\theta t) u\left(c_{t}\right) d t+\int_{s}^{\infty} \exp (-\theta t) u(\bar{c}) d t\right] \\
= & E_{0}\left\{\int_{0}^{s} \exp (-\theta t)\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t+\int_{s}^{\infty} \exp (-\theta t)\left[u\left(\hat{c}_{t}\right)-u(\bar{c})\right] d t\right\}>0
\end{aligned}
$$

because $c_{t}<c_{t}+b_{t}$ and $\bar{c}<\hat{c}_{t}$.
Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small $\gamma$ is examined.

Lemma A1: If $\gamma(0<\gamma<\infty)$ is sufficiently small, then $E_{0}($ Jalone $)-E_{0}($ NJtogether $)>0$.
Proof: $\lim _{\gamma \rightarrow 0}\left[E_{0}(\right.$ Jalone $)-E_{0}($ NJtogether $\left.)\right]$

$$
\begin{aligned}
& =E_{0} \int_{0}^{s} \exp (-\theta t) \lim _{\gamma \rightarrow 0}\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t+E_{0} \int_{s}^{\infty} \exp (-\theta t) \lim _{\gamma \rightarrow 0}[u(\bar{c}-\bar{a})-u(\bar{c})] d t \\
& =E_{0} \int_{0}^{s} \exp (-\theta t) b_{t} d t-E_{0} \int_{s}^{\infty} \exp (-\theta t) \bar{a} d t \\
& =E_{0} \int_{0}^{s} \exp (-\theta t) b_{t} d t-E_{0} \theta\left[\int_{0}^{s}\left(b_{r} \exp \int_{r}^{s} i_{q} d q\right) d r\right] \int_{s}^{\infty} \exp (-\theta t) d t \\
& =E_{0} \int_{0}^{s} \exp (-\theta t) b_{t} d t-E_{0} \exp (-\theta s) \int_{0}^{s}\left(b_{r} \exp \int_{r}^{s} i_{q} d q\right) d r \\
& \left.=E_{0} \exp (-\theta s) \int_{0}^{s} b_{t} b_{t} \exp [\theta(s-t)]-\exp \int_{t}^{s} i_{q} d q\right\} d t>0,
\end{aligned}
$$

[^6]because, if $0 \leq t<s$, then $i_{t}<\theta$ and $\exp [\theta(s-t)]>\exp \int_{t}^{s} i_{q} d q$. Hence, because $\exp [\theta(s-t)]$ $>\exp \int_{t}^{s} i_{q} d q, E_{0}($ Jalone $)-E_{0}($ NJtogether $)>0$ for sufficiently small $\gamma$.

Second, the opposite special case (i.e., a utility function with a sufficiently large $\gamma$ ) is examined.

Lemma A2: If $\gamma(0<\gamma<\infty)$ is sufficiently large and if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$, then $E_{0}$ (Jalone)$E_{0}($ NJtogether $)<0$.
Proof: Because $0<b_{t}$, then

$$
\lim _{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c} 1-\gamma}\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right]=\lim _{\gamma \rightarrow \infty}\left[\left(\frac{c_{t}+b_{t}}{\bar{c}}\right)^{1-\gamma}-\left(\frac{c_{t}}{\bar{c}}\right)^{1-\gamma}\right]=0
$$

for any period $t(<s)$. On the other hand, because $0<\bar{a}$, then for any period $t(<s)$, if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$,

$$
\lim _{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}}[u(\bar{c}-\bar{a})-u(\bar{c})]=\lim _{\gamma \rightarrow \infty}\left[\left(1-\frac{\bar{a}}{\bar{c}}\right)^{1-\gamma}-1\right]=\infty .
$$

Thus,

$$
\begin{aligned}
& \lim _{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}}\left[E_{0}(\text { Jalone })-E_{0}(\text { NJtogether })\right] \\
= & \lim _{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_{0}^{s} \exp (-\theta t) \lim _{\gamma \rightarrow \infty}\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t \\
+ & \lim _{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_{s}^{\infty} \exp (-\theta t) \lim _{\gamma \rightarrow \infty}[u(\bar{c}-\bar{a})-u(\bar{c})] d t \\
= & 0+\infty>0 .
\end{aligned}
$$

Because $\frac{1-\gamma}{\bar{c}^{1-\gamma}}<0$ for any $\gamma(1<\gamma<\infty)$, then if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1, E_{0}$ (Jalone) $-E_{0}$ (NJtogether) $<0$ for sufficiently large $\gamma(<\infty)$.

The condition $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$ indicates that path $N J$ from $c_{0}$ to $\bar{c}$ deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state $\bar{c}$ not taking much time. Because steady states are irrelevant to the degree of risk aversion $(\gamma)$, both $c_{0}$ and $\bar{c}$ are irrelevant to $\gamma$.

By Lemmas A1 and A2, it can be proved that $E_{0}($ Jalone $)-E_{0}($ NJtogether $)<0$ is possible.

Lemma A3: If $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$, then there is a $\gamma^{*}\left(0<\gamma^{*}<\infty\right)$ such that if $\gamma^{*}<\gamma<\infty$,
$E_{0}($ Jalone $)-E_{0}($ NJtogether $)<0$.
Proof: If $\gamma(>0)$ is sufficiently small, then $E_{0}($ Jalone $)-E_{0}($ NJtogether $)>0$ by Lemma A1, and if $\gamma(<\infty)$ is sufficiently large and if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$, then $E_{0}$ (Jalone) $-E_{0}$ (NJtogether) $<0$ by Lemma A2. Hence, if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$, there is a certain $\gamma^{*}\left(0<\gamma^{*}<\infty\right)$ such that, if $\gamma^{*}<\gamma<\infty$, then $E_{0}($ Jalone $)-E_{0}($ NJtogether $)<0$.

However, $E_{0}($ Jtogether $)-E_{0}($ NJalone $)>0$ because both Jtogether and NJalone indicate that all the other households choose option $J$; thus, the values of $i_{t}$ and $k_{t}$ are the same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these $i_{t}$ and $k_{t}$, deviating alone from the Pareto efficient path (NJalone) gives a lower expected utility than Jtogether to the $N J$ household. Both Jalone and NJtogether indicate that all the other households choose option $N J$ and $i_{t}$ and $k_{t}$ are not those of the Pareto efficient path. Hence, the sign of $E_{0}($ Jalone $)-E_{0}($ NJtogether $)$ varies depending on the conditions, as Lemma A3 indicates.

By Lemma A3 and the property $E_{0}($ Jtogether $)-E_{0}($ NJalone $)>0$, the possibility of the choice of a Pareto inefficient transition path, that is, $E_{0}(J)-E_{0}(N J)<0$, is shown.

Proposition A1: If $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$ and $\gamma^{*}<\gamma<\infty$, then there is a $p^{*}\left(0 \leq p^{*} \leq 1\right)$ such that if $p=p^{*}, E_{0}(J)-E_{0}(N J)=0$, and if $p<p^{*}, E_{0}(J)-E_{0}(N J)<0$.
Proof: By Lemma A3, if $\gamma^{*}<\gamma<\infty$, then $E_{0}($ Jalone $)-E_{0}($ NJtogether $)<0$ and $E_{0}($ Jtogether $)-E_{0}($ NJalone $)>0$. By equations (A1) and (A2),

$$
E_{0}(J)-E_{0}(N J)=p\left[E_{0}(\text { Jtogether })-E_{0}(\text { NJalone })\right]+(1-p)\left[E_{0}(\text { Jalone })-E_{0}(\text { NJtogether })\right] .
$$

Thus, if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1$ and $\gamma^{*}<\gamma<\infty, \lim _{p \rightarrow 0}\left[E_{0}(J)-E_{0}(N J)\right]=E_{0}($ Jalone $)-E_{0}($ NJtogether $)<0$ and $\lim _{p \rightarrow 1}\left[E_{0}(J)-E_{0}(N J)\right]=E_{0}($ Jtogether $)-E_{0}($ NJalone $)>0$. Hence, by the intermediate value theorem, there is $p^{*}\left(0 \leq p^{*} \leq 1\right)$ such that if $p=p^{*}, E_{0}(J)-E_{0}(N J)=0$ and if $p<p^{*}$, $E_{0}(J)-E_{0}(N J)<0$.

Proposition A1 indicates that, if $0<\lim _{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}<1, \gamma^{*}<\gamma<\infty$, and $p<p^{*}$, then the choice of option $N J$ gives the higher expected utility than that of option $J$ to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, so a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against the conjecture that households never rationally choose a Pareto inefficient transition path in a frictionless economy.

## A2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them, $\gamma^{*}<\gamma<\infty$ may appear rather strict. If $\gamma^{*}$ is very large, path $N J$ will rarely be chosen. However, if path $N J$ is such that consumption is reduced sharply after the shock, the $N J$ option yields a higher expected utility than the $J$
option even though $\gamma$ is very small. For example, for any $\gamma(0<\gamma<\infty)$,

$$
\begin{aligned}
& \lim _{s \rightarrow 0} \frac{1}{s}\left[E_{0}(\text { Jalone })-E_{0}(\text { NJtogether })\right] \\
= & \lim _{s \rightarrow 0} \frac{1}{s} \int_{0}^{s} \exp (-\theta t)\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t+\lim _{s \rightarrow 0} \frac{1}{s} \int_{s}^{\infty} \exp (-\theta t)[u(\bar{c}-\bar{a})-u(\bar{c})] d t \\
= & u\left(c_{0}+b_{0}\right)-u\left(c_{0}\right)-\frac{1}{\theta} \lim _{s \rightarrow 0} \frac{u(\bar{c})-u\left(\bar{c}-s \theta b_{0}\right)}{s}=u\left(c_{0}+b_{0}\right)-u\left(c_{0}\right)-b_{0} \frac{d u(\bar{c})}{d \bar{c}} \\
= & \frac{\left(c_{0}+b_{0}\right)^{1-\gamma}-c_{0}^{1-\gamma}}{1-\gamma}-b_{0} \bar{c}^{-\gamma}=\bar{c}^{-\gamma}\left\{\bar{c}^{\gamma}\left[\frac{\left(c_{0}+b_{0}\right)^{1-\gamma}}{1-\gamma}-\frac{c_{0}^{1-\gamma}}{1-\gamma}\right]-b_{0}\right\}<0,
\end{aligned}
$$

because $\lim _{\gamma \rightarrow 1} \bar{c} \gamma\left[\frac{\left(c_{0}+b_{0}\right)^{1-\gamma}}{1-\gamma}-\frac{c_{0}^{1-\gamma}}{1-\gamma}\right]=\bar{c}\left[\ln \left(c_{0}+b_{0}\right)-\ln \left(c_{0}\right)\right]=\bar{c} \ln \left(1+\frac{b_{0}}{c_{0}}\right)<b_{0}$ and $\lim _{\gamma \rightarrow \infty} \bar{c}^{\gamma}\left[\frac{\left(c_{0}+b_{0}\right)^{1-\gamma}}{1-\gamma}-\frac{c_{0}^{1-\gamma}}{1-\gamma}\right]=\lim _{\gamma \rightarrow \infty} \bar{c}^{\gamma} c_{0}^{1-\gamma}\left[\frac{\left(1+\frac{b_{0}}{c_{0}}\right)^{1-\gamma}-1}{1-\gamma}\right]=0$ because $\bar{c}<c_{0}$. That is, for each combination of path $N J$ and $\gamma$, there is $s^{*}(>0)$ such that, if $s<s^{*}$, then $E_{0}$ (Jalone) $-E_{0}($ NJtogether $)<0$.

Consider an example in which path $N J$ is such that $b_{t}$ is constant and $b_{t}=\bar{b}$ before $s$ (Figure A3); thus, $E_{0} \int_{0}^{s} b_{t}=s \bar{b}$. In this $N J$ path, consumption is reduced more sharply than it is in the case shown in Figure A2. In this case, because $\bar{a}>E_{0} \theta \int_{0}^{s} b_{t}=\theta s \bar{b}, 0<\gamma$, and $c_{s}<c_{t}$ for $t<s$ then $E_{0} \int_{0}^{s} \exp (-\theta t)\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t<E_{0} \int_{0}^{s} \exp (-\theta t) d t\left[u\left(c_{s}+\bar{b}\right)-u\left(c_{s}\right)\right]=$ $E_{0} \frac{1-\exp (-\theta s)}{\theta}\left[u\left(c_{s}+\bar{b}\right)-u\left(c_{s}\right)\right]$, and in addition, $E_{0} \int_{s}^{\infty} \exp (-\theta t)[u(\bar{c}-\bar{a})-u(\bar{c})] d t=$ $E_{0} \int_{s}^{\infty} \exp (-\theta t) d t[u(\bar{c}-\bar{a})-u(\bar{c})]=E_{0} \frac{\exp (-\theta s)}{\theta}[u(\bar{c}-\bar{a})-u(\bar{c})]<E_{0} \frac{\exp (-\theta s)}{\theta}[u(\bar{c}-\theta s \bar{b})-u(\bar{c})]$. Hence,

$$
\begin{aligned}
& E_{0}(\text { Jalone })-E_{0}(\text { NJtogether }) \\
= & E_{0} \int_{0}^{s} \exp (-\theta t)\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t+E_{0} \int_{s}^{\infty} \exp (-\theta t)[u(\bar{c}-\bar{a})-u(\bar{c})] d t \\
< & E_{0} \frac{1-\exp (-\theta s)}{\theta}\left[u\left(c_{s}+\bar{b}\right)-u\left(c_{s}\right)\right]+E_{0} \frac{\exp (-\theta s)}{\theta}[u(\bar{c}-\theta s \bar{b})-u(\bar{c})] \\
= & E_{0} \frac{1-\exp (-\theta s)}{\theta}\left\{\left[u\left(c_{s}+\bar{b}\right)-u\left(c_{s}\right)\right]-\frac{\exp (-\theta s)}{1-\exp (-\theta s)}[u(\bar{c})-u(\bar{c}-\theta s \bar{b})]\right\} .
\end{aligned}
$$

As $\gamma$ increases, the ratio $\frac{u\left(c_{s}+\bar{b}\right)-u\left(c_{s}\right)}{u(\bar{c})-u(\bar{c}-\theta s \bar{b})}$ decreases; thus, larger values of $s$ can satisfy $E_{0}($ Jalone $)-E_{0}($ NJtogether $)<0$. For example, suppose that $\bar{c}=10, c_{s}=10.2, \bar{b}=0.3$, and $\theta$
$=0.05$. If $\gamma=1$, then $s^{*}=1.5$ at the minimum, and if $\gamma=5$, then $s^{*}=6.8$ at the minimum. This result implies that, if option $N J$ is such that consumption is reduced relatively sharply after the shock (e.g., $b_{t}=\bar{b}$ ) and $p<p^{*}$, option $N J$ will usually be chosen. Choosing option $N J$ is not a special case observed only if $\gamma$ is very large, but option $N J$ can normally be chosen when the value of $\gamma$ is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a recession, consumption usually declines sharply after the shock, which suggests that households have chosen the $N J$ option.

## A3 Nash equilibrium

## A3.1 A Nash equilibrium consisting of $N J$ strategies

A household strategically determines whether to choose the $J$ or $N J$ option, considering other households' choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are $H(\in N)$ identical households in the economy where $H$ is sufficiently large (as assumed in Section A1). Let $q_{\eta}\left(0 \leq q_{\eta} \leq 1\right)$ be the probability that a household $\eta(\in H)$ chooses option $J$. The average utility of the other households almost equals that of all households because $H$ is sufficiently large. Hence, the average expected utilities of the other households that choose the $J$ and $N J$ options are $E_{0}($ Jtogether $)$ and $E_{0}($ NJtogether $)$, respectively. Hence, the payoff matrix of the $H$-dimensional symmetric mixed strategy game can be described as shown in Table A1. Each identical household determines its behavior on the basis of this payoff matrix.

In this mixed strategy game, the strategy profiles

$$
\left(q_{1}, q_{2}, \ldots, q_{H}\right)=\left\{(1,1, \ldots, 1),\left(p^{*}, p^{*}, \ldots, p^{*}\right),(0,0, \ldots, 0)\right\}
$$

are Nash equilibria for the following reason. By Proposition A1, the best response of household $\eta$ is $J$ (i.e., $q_{\eta}=1$ ) if $p>p^{*}$, indifferent between $J$ and $N J$ (i.e., any $q_{\eta} \in[0,1]$ ) if $p=p^{*}$, and $N J$ (i.e., $q_{\eta}=0$ ) if $p<p^{*}$. Because all households are identical, the best-response correspondence of each household is identical such that $q_{\eta}=1$ if $p>p^{*},[0,1]$ if $p=p^{*}$, and 0 if $p<p^{*}$ for any household $\eta \in H$. Hence, the mixed strategy profiles $(1,1, \ldots, 1),\left(p^{*}, p^{*}, \ldots, p^{*}\right)$, and $(0,0, \ldots, 0)$ are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution ( $1,1, \ldots, 1$ ) (i.e., Jtogether) is a pure strategy Nash equilibrium, but a Pareto inefficient transition path $(0,0, \ldots, 0)$ (i.e., NJtogether) is also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium $\left(p^{*}, p^{*}, \ldots, p^{*}\right)$.

## A3.2 Selection of equilibrium

Determining which Nash equilibrium, either NJtogether $(0,0, \ldots, 0)$ or Jtogether $(1,1, \ldots, 1)$, is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose a very low $p$ and select the NJtogether $(0,0, \ldots, 0)$ equilibrium. Because

$$
\begin{aligned}
& E_{0}(\text { Jalone })-E_{0}(\text { NJalone }) \\
= & E_{0}\left\{\int_{0}^{s} \exp (-\theta t)\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t+\int_{s}^{\infty} \exp (-\theta t)\left[u(\bar{c}-\bar{a})-u\left(\hat{c}_{t}+a_{t}\right)\right] d t\right\} \\
< & E_{0}\left\{\int_{0}^{s} \exp (-\theta t)\left[u\left(c_{t}+b_{t}\right)-u\left(c_{t}\right)\right] d t+\int_{s}^{\infty} \exp (-\theta t)[u(\bar{c}-\bar{a})-u(\bar{c})] d t\right\}
\end{aligned}
$$

$$
\begin{equation*}
=E_{0}(\text { Jalone })-E_{0}(\text { NJtogether })<0, \tag{A7}
\end{equation*}
$$

by Lemma A3, Jalone is the worst choice in terms of the amount of payoff, followed by NJtogether, and NJalone, and Jtogether is the best. The outcomes of choosing option $J$ are more dispersed than those of option NJ. If households have a risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option ( $N J$ ), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Because all households are identical and know inequality (A7), all households will equally suppose that they all prefer the less dispersed $N J$ option; therefore, all of them will suppose a very low $p$, particularly $p=0$, and select the NJtogether $(0,0, \ldots, 0)$ equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and "animal spirits" (e.g., pessimism or optimism) are unnecessary to explain the selection.

## A4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products because of $b_{t}$. They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the $J$ path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock, unutilized products because of $b_{t}$ will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates unutilized products because of $b_{t}$ will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period, unutilized products are generated and eliminated, but in the next period, another, new, unutilized products are generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium.

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Figure A1: A time preference shock


Figure A2: The paths of Jalone and NJalone


Figure A3: A Pareto inefficient transition path


## Table A1 The payoff matrix

|  |  | Any other household |  |
| :---: | :---: | :---: | :---: |
|  |  | $J$ | $N J$ |
| $\begin{aligned} & 4 \\ & \overrightarrow{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $J$ | $E_{0}\left(\right.$ Jtogether), ${ }_{0}$ (Jtogether) | $E_{0}($ Jalone $)$, E0(NJtogether) |
|  | $N J$ | $E_{0}($ NJalone $)$, $\mathrm{E}_{0}($ Jtogether $)$ | $E_{0}$ (NJtogether), $E_{0}$ (NJtogether) |


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[^1]:    ${ }^{1}$ If depreciation of capital is assumed to exist, the "consumption" of excess capital will be achieved by a reduction of investments that correspond to depreciated capital and an increase in consumer goods and services.

[^2]:    ${ }^{2}$ Some economists argue the reverse causation from growth to institutional improvement (e.g., Barro, 1999) or that institutional improvement has a smaller impact on growth than human capital (Glaeser et al., 2004).

[^3]:    ${ }^{3}$ If capital depreciation is assumed to exist, consumption of excess capital will be achieved by a reduction of investments that corresponds to depreciated capital inputs and an increase in consumer goods and services.

[^4]:    ${ }^{4}$ More correctly, the economy never arrives exactly at the posterior steady state, but it arrives close to it in a definite period.

[^5]:    5 The model in Appendix is based on the model by Harashima (2012). See also Harashima (2004, 2013b).

[^6]:    ${ }^{6}$ The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004).

